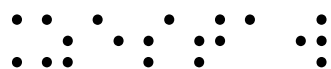
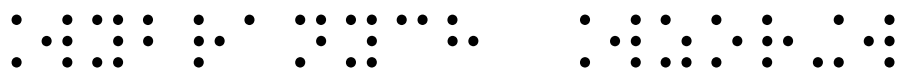


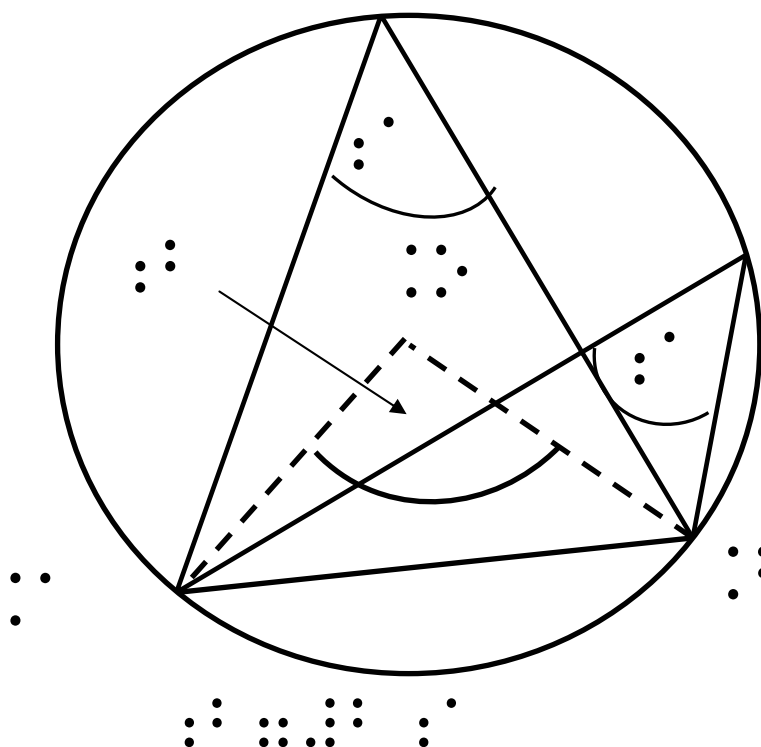
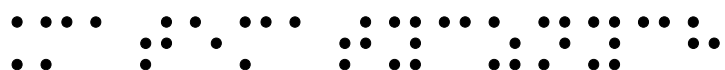
ZESTAW



WYBRANYCH WZORÓW



MATEMATYCZNYCH

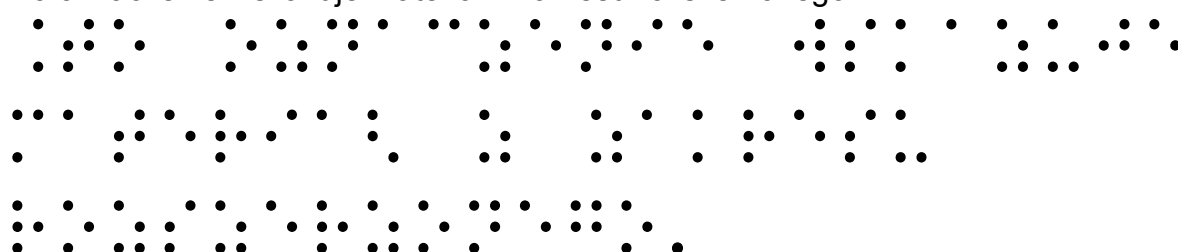


Materiały pomocnicze opracowane dla potrzeb egzaminu maturalnego i dopuszczone jako pomoce egzaminacyjne.

Zestaw matematycznych wzorów został przygotowany dla potrzeb egzaminu maturalnego z matematyki. Zestaw ten został opracowany w Centralnej Komisji Egzaminacyjnej we współpracy z pracownikami wyższych uczelni oraz w konsultacji z ekspertami z okręgowych komisji egzaminacyjnych.

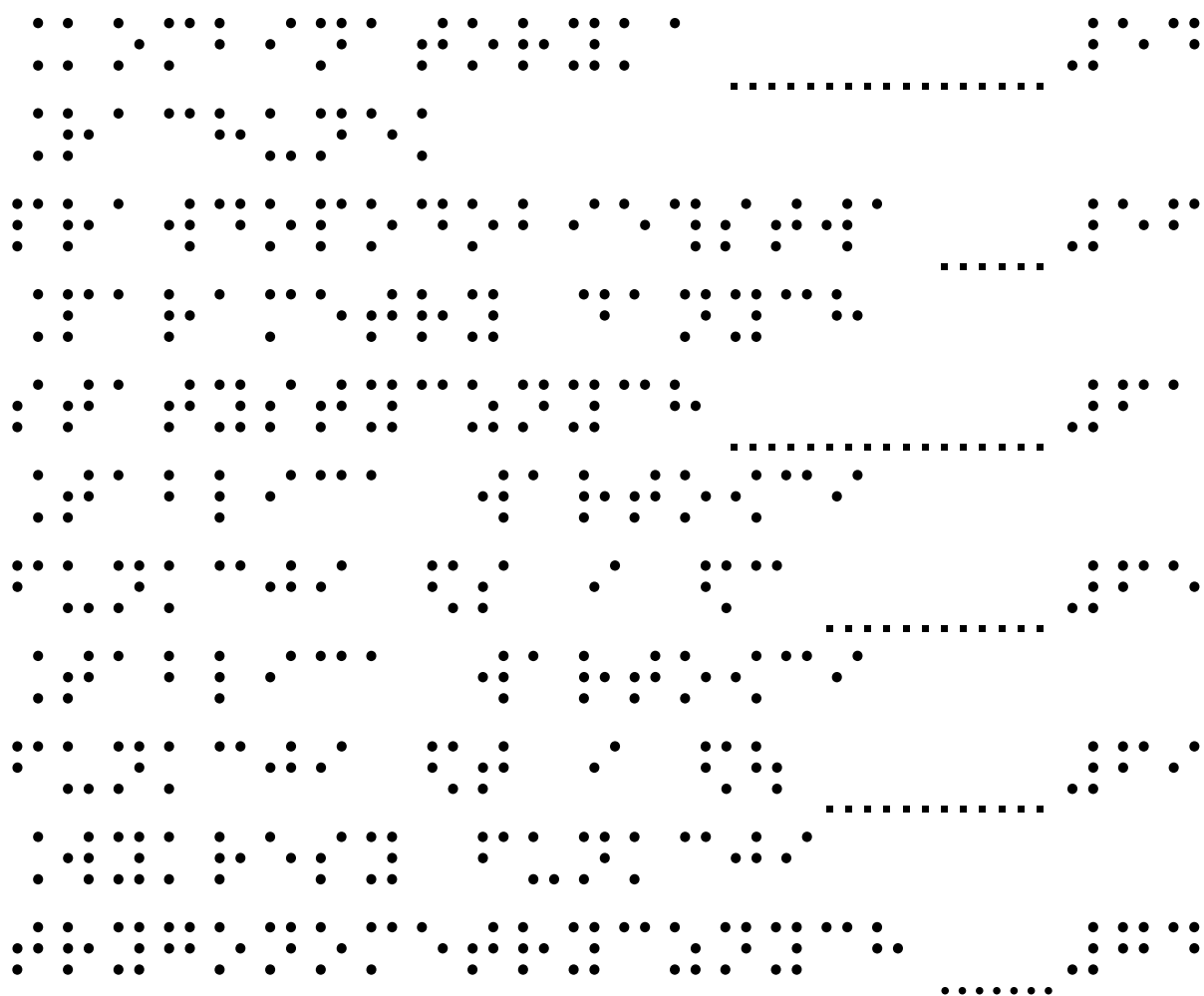
Na zlecenie CKE zestaw wzorów matematycznych dla potrzeb egzaminu maturalnego z matematyki dla niewidomych i słabo widzących przystosowały mgr Bogumiła Golańska i mgr inż. Agnieszka Lato w Specjalnym Ośrodku Szkolno Wychowawczym dla Dzieci Niewidomych i Słabowidzących, Kraków ul. Tyniecka 7.

To oznaczenie wskazuje materiał z zakresu rozszerzonego.



SPIS TREŚCI

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100	100



A 15x15 grid of black dots on a white background. The dots are arranged in a sparse, abstract pattern, forming a large, irregular shape that resembles a stylized letter 'A' or a similar character. The pattern is composed of many small, black dots, some of which are clustered together, while others are isolated. The overall effect is a minimalist, high-contrast graphic.



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Silnia. Symbol Newtona

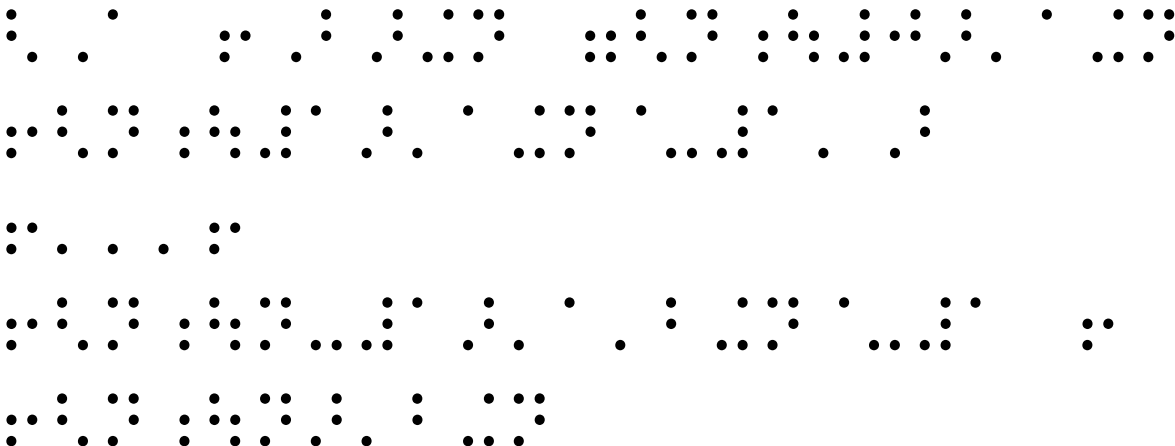
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Wszystkie te rzeczy są dla ciebie, a ty jesteś dla wszystkich.

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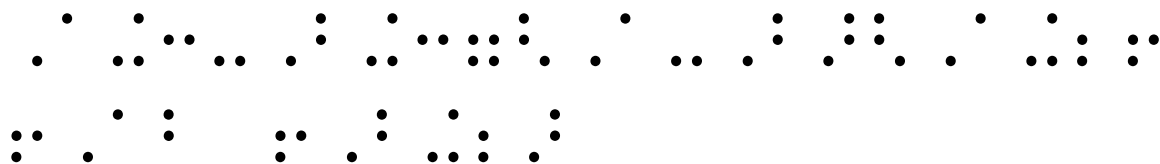
Dwumian Newtona

Wszystkie te rzeczy są dla ciebie, a ty jesteś dla wszystkich.
Wszystkie te rzeczy są dla ciebie, a ty jesteś dla wszystkich.
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Wszystkie te rzeczy są dla ciebie, a ty jesteś dla wszystkich.



Wzory skróconego mnożenia

$$\begin{aligned}
 & (a+b)^2 = a^2 + 2ab + b^2 \\
 & (a-b)^2 = a^2 - 2ab + b^2 \\
 & (a+b)(a-b) = a^2 - b^2 \\
 & (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \\
 & (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \\
 & (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \\
 & (a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4 \\
 & (a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 \\
 & (a-b)^5 = a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5 \\
 & (a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6 \\
 & (a-b)^6 = a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6 \\
 & (a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7 \\
 & (a-b)^7 = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7 \\
 & (a+b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8 \\
 & (a-b)^8 = a^8 - 8a^7b + 28a^6b^2 - 56a^5b^3 + 70a^4b^4 - 56a^3b^5 + 28a^2b^6 - 8ab^7 + b^8 \\
 & (a+b)^9 = a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9 \\
 & (a-b)^9 = a^9 - 9a^8b + 36a^7b^2 - 84a^6b^3 + 126a^5b^4 - 126a^4b^5 + 84a^3b^6 - 36a^2b^7 + 9ab^8 - b^9 \\
 & (a+b)^{10} = a^{10} + 10a^9b + 45a^8b^2 + 120a^7b^3 + 210a^6b^4 + 252a^5b^5 + 210a^4b^6 + 120a^3b^7 + 45a^2b^8 + 10ab^9 + b^{10} \\
 & (a-b)^{10} = a^{10} - 10a^9b + 45a^8b^2 - 120a^7b^3 + 210a^6b^4 - 252a^5b^5 + 210a^4b^6 - 120a^3b^7 + 45a^2b^8 - 10ab^9 + b^{10}
 \end{aligned}$$



Ciagi

A 6x6 grid of 36 dot patterns, each representing a unique combination of the six basic dot patterns from the previous block. The patterns are arranged in a 6x6 grid, with each row and column containing a different combination of the six basic patterns.

A 10x10 grid of black dots on a white background. The dots are arranged in a pattern that suggests a network or graph structure, with clusters of dots and connecting lines. The dots are arranged in a way that suggests a network or graph structure, with clusters of dots and connecting lines.

• • • • •

The image displays a 4x10 grid of 40 small black dot patterns, each representing a different character from the 96-character set of the Braille English Braille alphabet. The patterns are arranged in four rows and ten columns, showing a variety of dot configurations used in Braille notation.

A 3x10 grid of black dots representing a sparse matrix. The dots are arranged in a pattern that suggests a banded structure with some off-diagonal elements.

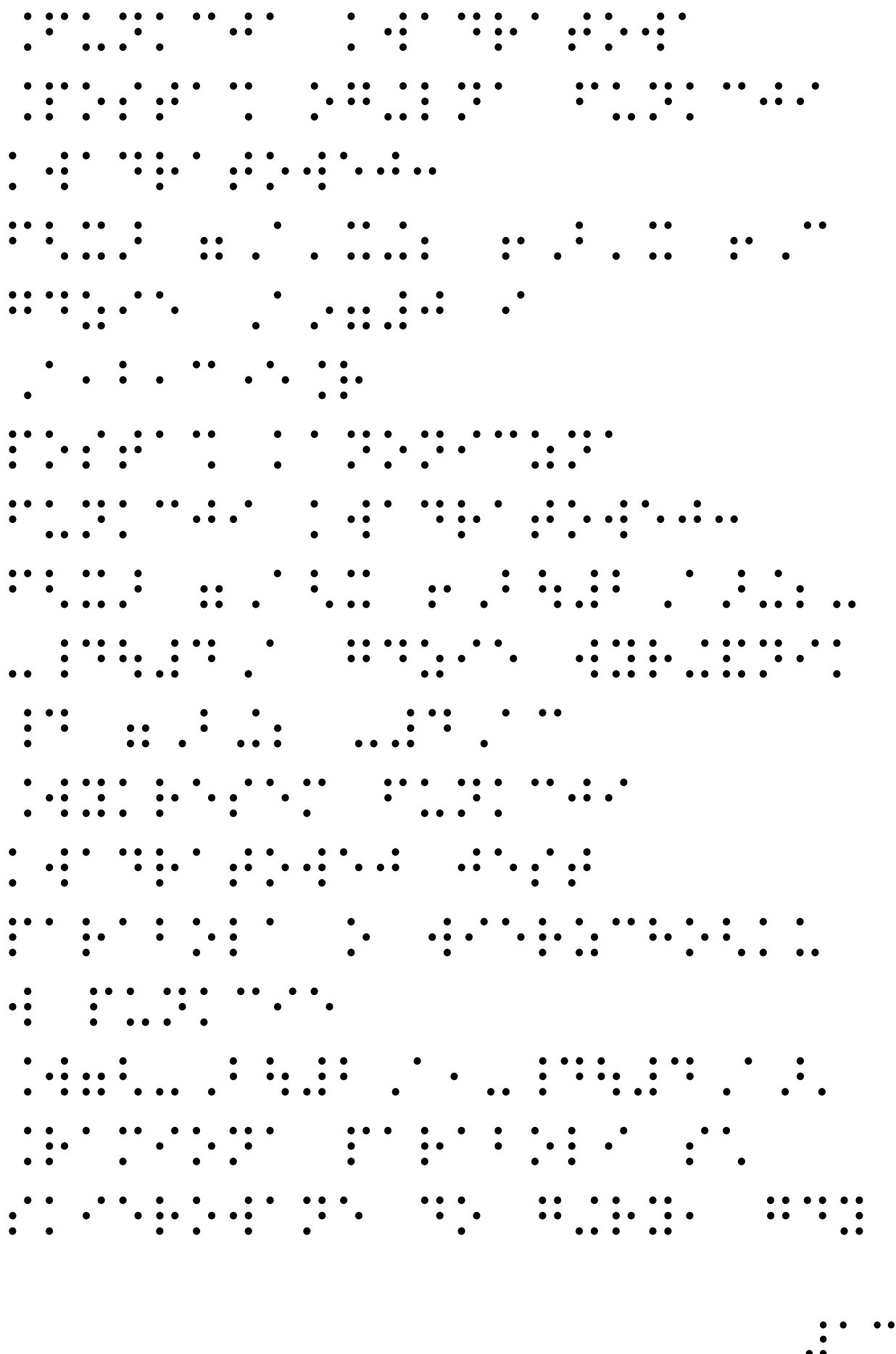
A 3x10 grid of black dots representing a sparse matrix. The dots are arranged in a pattern that suggests a banded structure with some off-diagonal elements.

The image shows three distinct dot patterns. The first pattern is a 2x2 square of dots. The second pattern is a 2x2 square of dots with a single dot centered below it. The third pattern is a 2x2 square of dots with a single dot centered to its right.

The image shows three 5x5 dot patterns. The first pattern represents the digit '1' with dots at (1,1), (1,3), (1,5), (2,2), (2,4), (3,3), (4,2), (4,4), (5,1), and (5,5). The second pattern represents the digit '2' with dots at (1,3), (1,5), (2,1), (2,3), (2,5), (3,1), (3,3), (3,5), (4,1), and (4,3). The third pattern represents the digit '3' with dots at (1,1), (1,3), (1,5), (2,1), (2,3), (2,5), (3,1), (3,3), (3,5), (4,1), and (4,3).

Figure 1 consists of eight scatter plots arranged in a 2x4 grid. The top row represents the year 2000, and the bottom row represents the year 2005. Each plot shows the frequency of families with a certain number of children (0 to 5). The x-axis is labeled 'Number of children' and the y-axis is labeled 'Frequency'. The distributions are roughly bell-shaped, peaking at 2 children.

Funkcja kwadratowa



A 15x15 grid of black dots on a white background. The dots are arranged in a sparse, abstract pattern. The pattern is denser in some areas and sparser in others, with a general horizontal orientation. The dots form a stylized, fragmented image, possibly a face or a landscape feature, though the interpretation is subjective. The pattern is composed of several distinct clusters of dots, some of which are arranged in a way that suggests a stylized, fragmented image, possibly a face or a landscape feature, though the interpretation is subjective. The dots are arranged in a way that suggests a stylized, fragmented image, possibly a face or a landscape feature, though the interpretation is subjective. The pattern is composed of several distinct clusters of dots, some of which are arranged in a way that suggests a stylized, fragmented image, possibly a face or a landscape feature, though the interpretation is subjective.

Pochodna funkcji

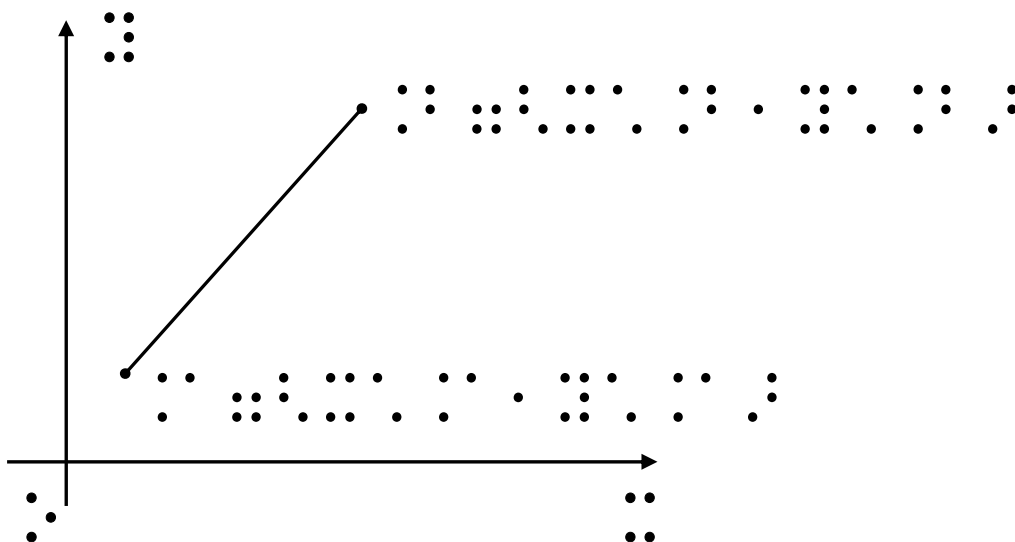
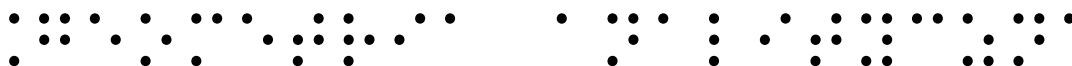
$f(x) = x^2 + 3x - 5$
 $f'(x) = 2x + 3$
 $f''(x) = 2$
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 $f^{(4)}(x) = 0$
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 $f^{(100)}(x) = 0$

1. Wzrost i masa ciała
 2. Ciężar ciała
 3. Ciężar ciała
 4. Ciężar ciała
 5. Ciężar ciała
 6. Ciężar ciała
 7. Ciężar ciała
 8. Ciężar ciała

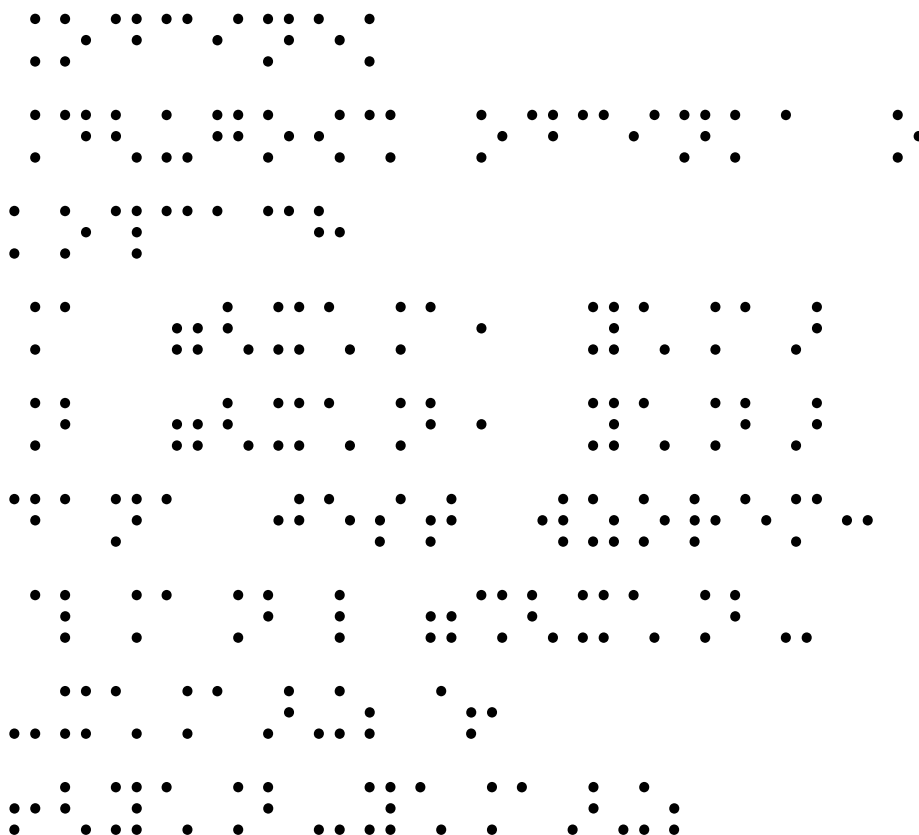
Równanie stycznej

1. Wzrost i masa ciała
 2. Ciężar ciała
 3. Ciężar ciała
 4. Ciężar ciała
 5. Ciężar ciała
 6. Ciężar ciała
 7. Ciężar ciała
 8. Ciężar ciała

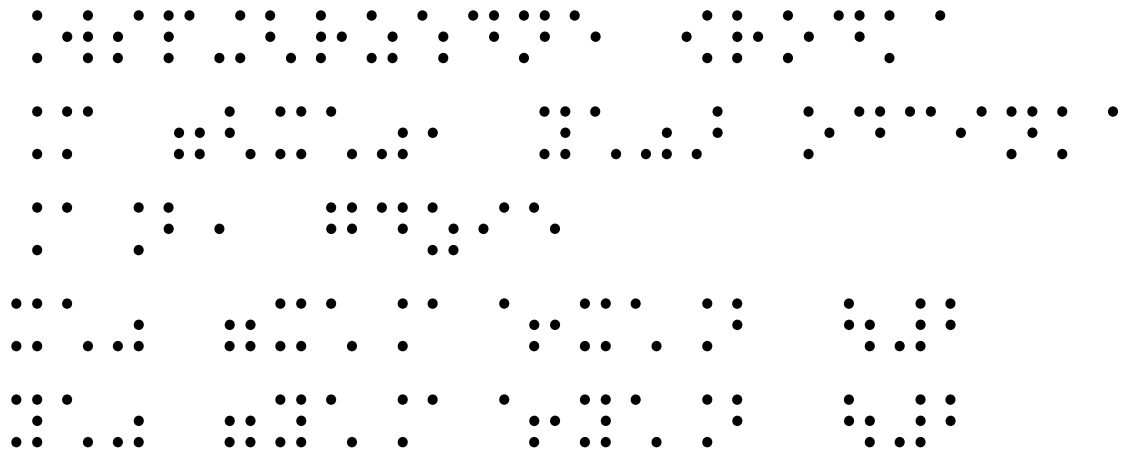
Geometria analityczna



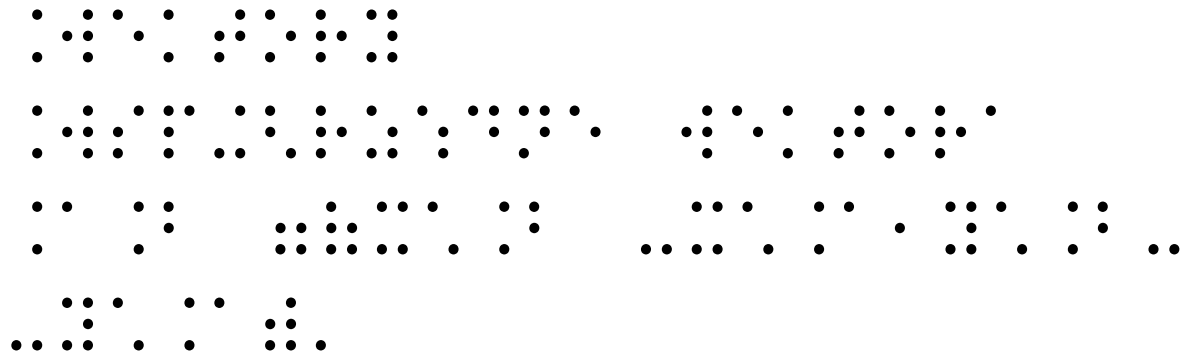
Odcinek



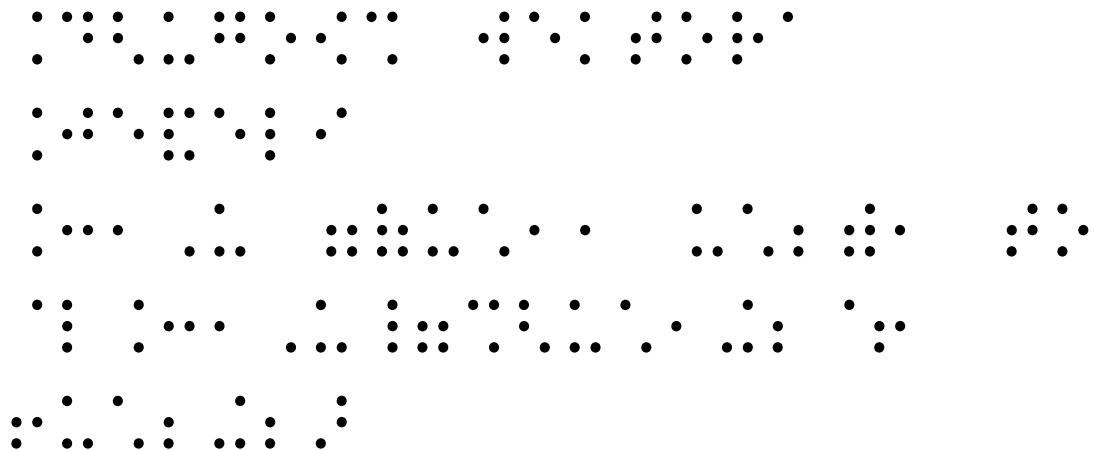
Środek odcinka

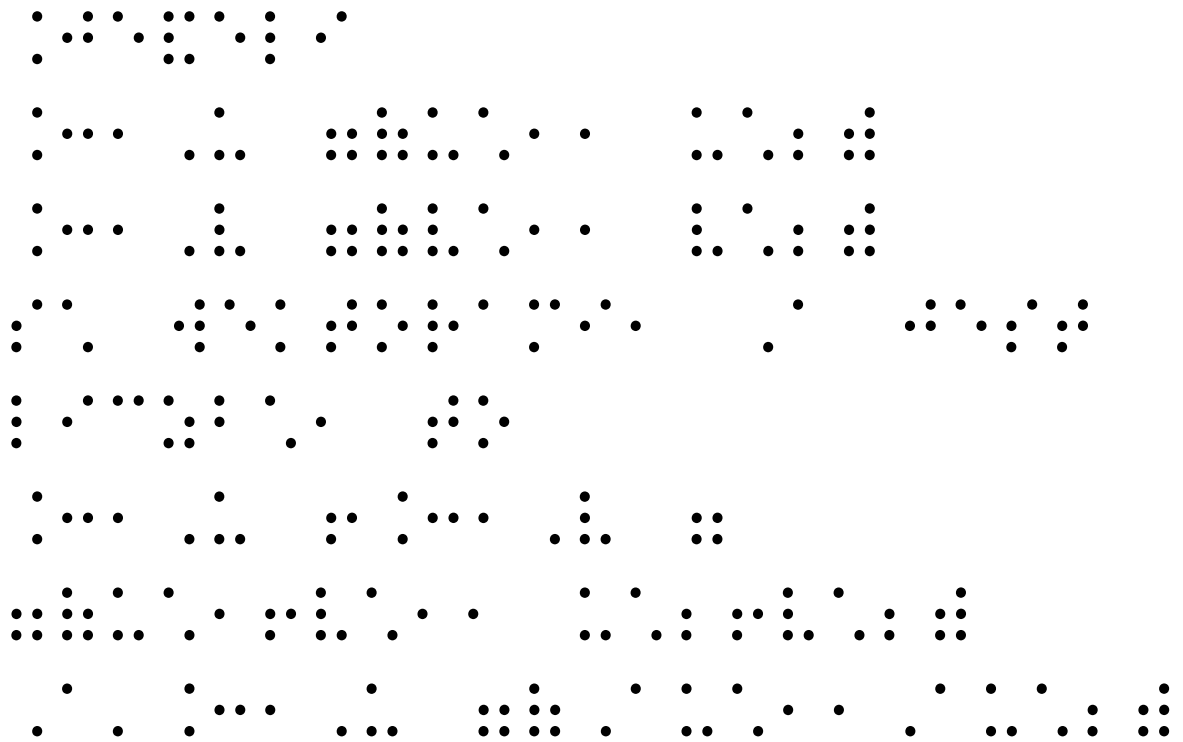


Wektory

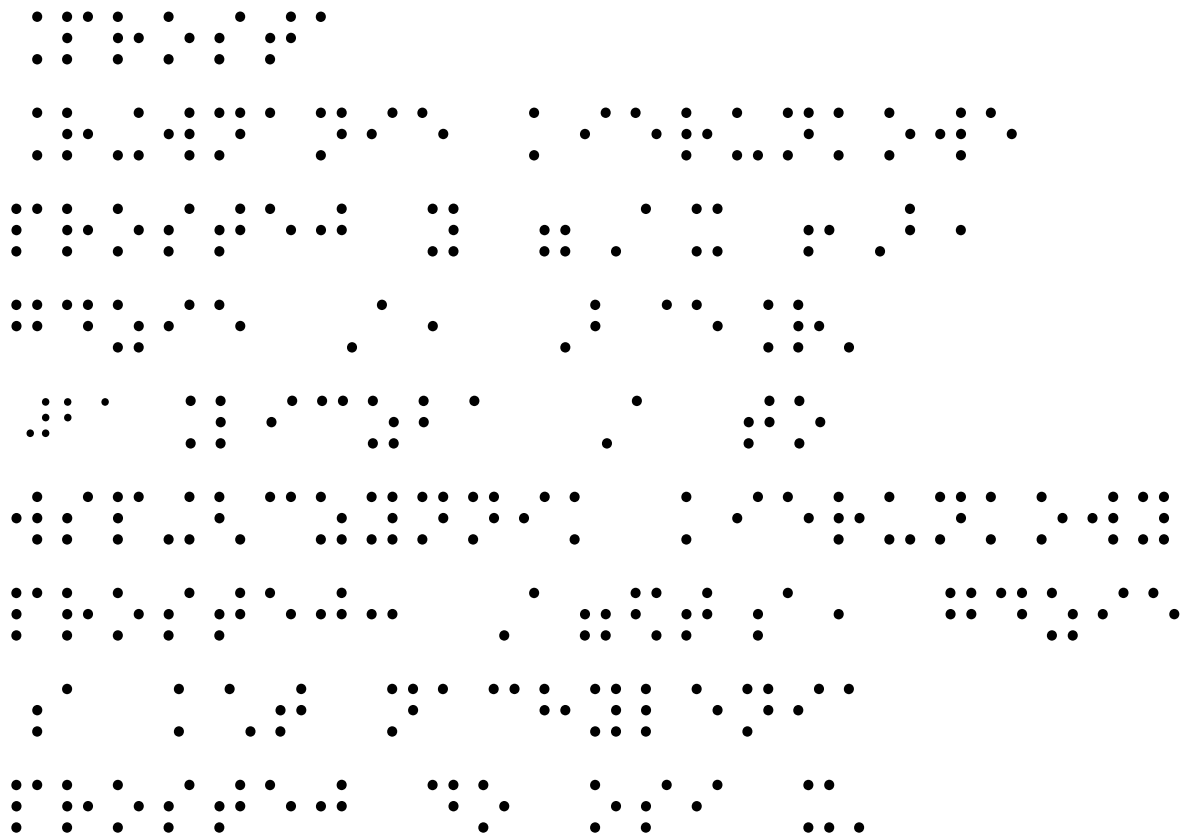


Długość wektora

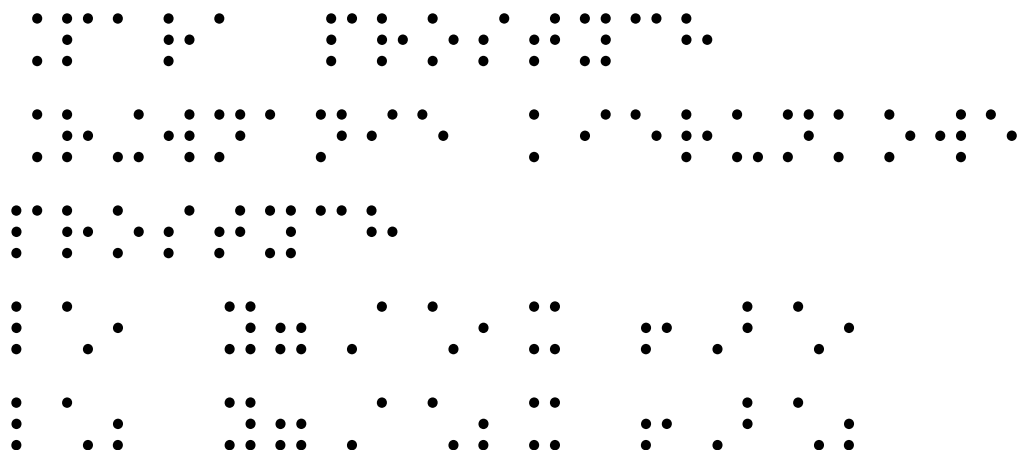




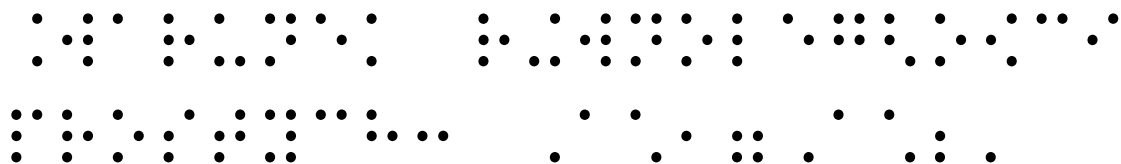
Prosta



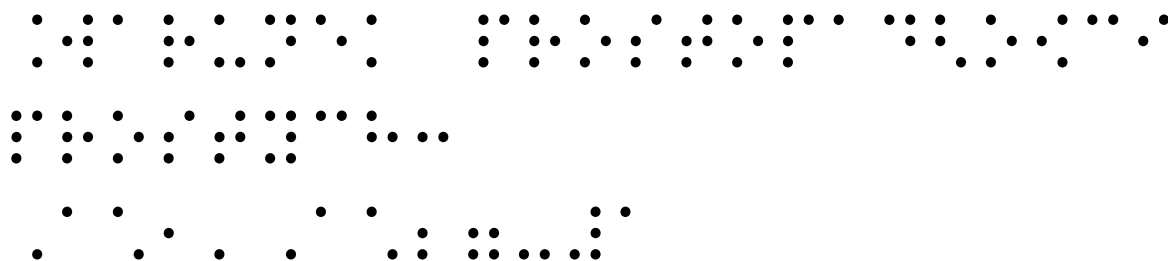
Para prostych



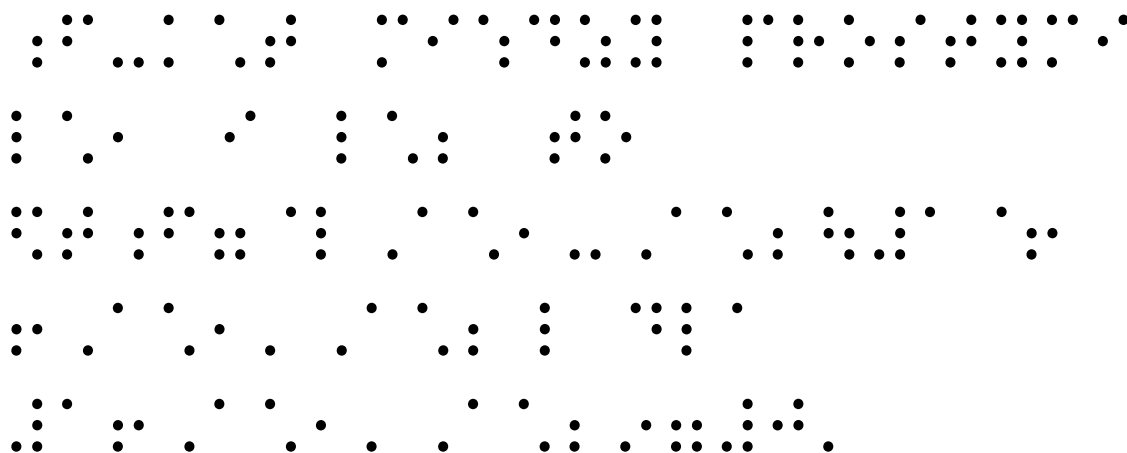
Warunek równoległości



Warunek prostopadłości



Kąt między prostymi



$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$
 $\vec{d} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix}$, $\vec{e} = \begin{pmatrix} 13 \\ 14 \\ 15 \end{pmatrix}$, $\vec{f} = \begin{pmatrix} 16 \\ 17 \\ 18 \end{pmatrix}$
 $\vec{g} = \begin{pmatrix} 19 \\ 20 \\ 21 \end{pmatrix}$, $\vec{h} = \begin{pmatrix} 22 \\ 23 \\ 24 \end{pmatrix}$, $\vec{i} = \begin{pmatrix} 25 \\ 26 \\ 27 \end{pmatrix}$
 $\vec{j} = \begin{pmatrix} 28 \\ 29 \\ 30 \end{pmatrix}$, $\vec{k} = \begin{pmatrix} 31 \\ 32 \\ 33 \end{pmatrix}$, $\vec{l} = \begin{pmatrix} 34 \\ 35 \\ 36 \end{pmatrix}$

Warunek równoległości

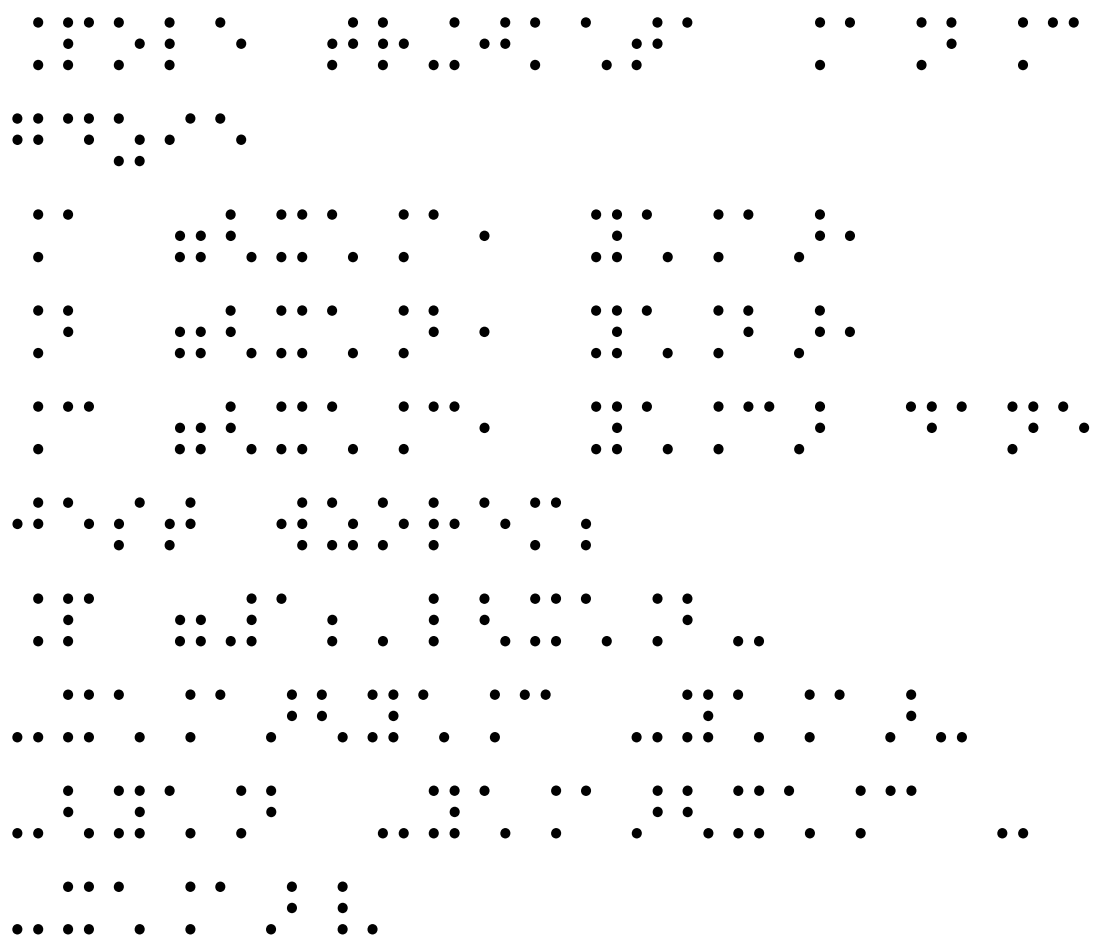
$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$, $\vec{d} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix}$
 $\vec{e} = \begin{pmatrix} 13 \\ 14 \\ 15 \end{pmatrix}$, $\vec{f} = \begin{pmatrix} 16 \\ 17 \\ 18 \end{pmatrix}$, $\vec{g} = \begin{pmatrix} 19 \\ 20 \\ 21 \end{pmatrix}$
 $\vec{h} = \begin{pmatrix} 22 \\ 23 \\ 24 \end{pmatrix}$, $\vec{i} = \begin{pmatrix} 25 \\ 26 \\ 27 \end{pmatrix}$, $\vec{j} = \begin{pmatrix} 28 \\ 29 \\ 30 \end{pmatrix}$

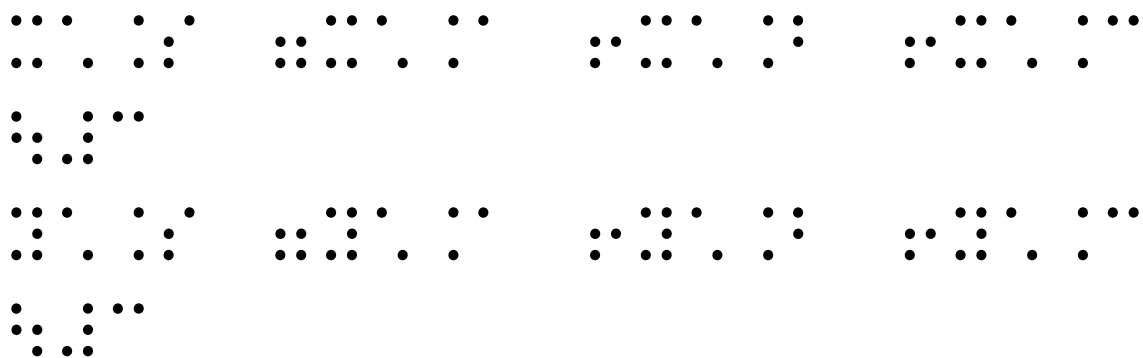
Warunek prostopadłości

$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$, $\vec{d} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix}$
 $\vec{e} = \begin{pmatrix} 13 \\ 14 \\ 15 \end{pmatrix}$, $\vec{f} = \begin{pmatrix} 16 \\ 17 \\ 18 \end{pmatrix}$, $\vec{g} = \begin{pmatrix} 19 \\ 20 \\ 21 \end{pmatrix}$
 $\vec{h} = \begin{pmatrix} 22 \\ 23 \\ 24 \end{pmatrix}$, $\vec{i} = \begin{pmatrix} 25 \\ 26 \\ 27 \end{pmatrix}$, $\vec{j} = \begin{pmatrix} 28 \\ 29 \\ 30 \end{pmatrix}$

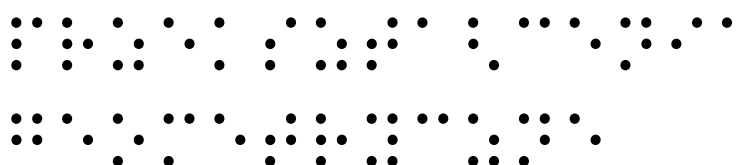
Kąt między prostymi

$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$, $\vec{d} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix}$
 $\vec{e} = \begin{pmatrix} 13 \\ 14 \\ 15 \end{pmatrix}$, $\vec{f} = \begin{pmatrix} 16 \\ 17 \\ 18 \end{pmatrix}$, $\vec{g} = \begin{pmatrix} 19 \\ 20 \\ 21 \end{pmatrix}$
 $\vec{h} = \begin{pmatrix} 22 \\ 23 \\ 24 \end{pmatrix}$, $\vec{i} = \begin{pmatrix} 25 \\ 26 \\ 27 \end{pmatrix}$, $\vec{j} = \begin{pmatrix} 28 \\ 29 \\ 30 \end{pmatrix}$
 $\vec{k} = \begin{pmatrix} 31 \\ 32 \\ 33 \end{pmatrix}$, $\vec{l} = \begin{pmatrix} 34 \\ 35 \\ 36 \end{pmatrix}$, $\vec{m} = \begin{pmatrix} 37 \\ 38 \\ 39 \end{pmatrix}$
 $\vec{n} = \begin{pmatrix} 40 \\ 41 \\ 42 \end{pmatrix}$, $\vec{o} = \begin{pmatrix} 43 \\ 44 \\ 45 \end{pmatrix}$, $\vec{p} = \begin{pmatrix} 46 \\ 47 \\ 48 \end{pmatrix}$

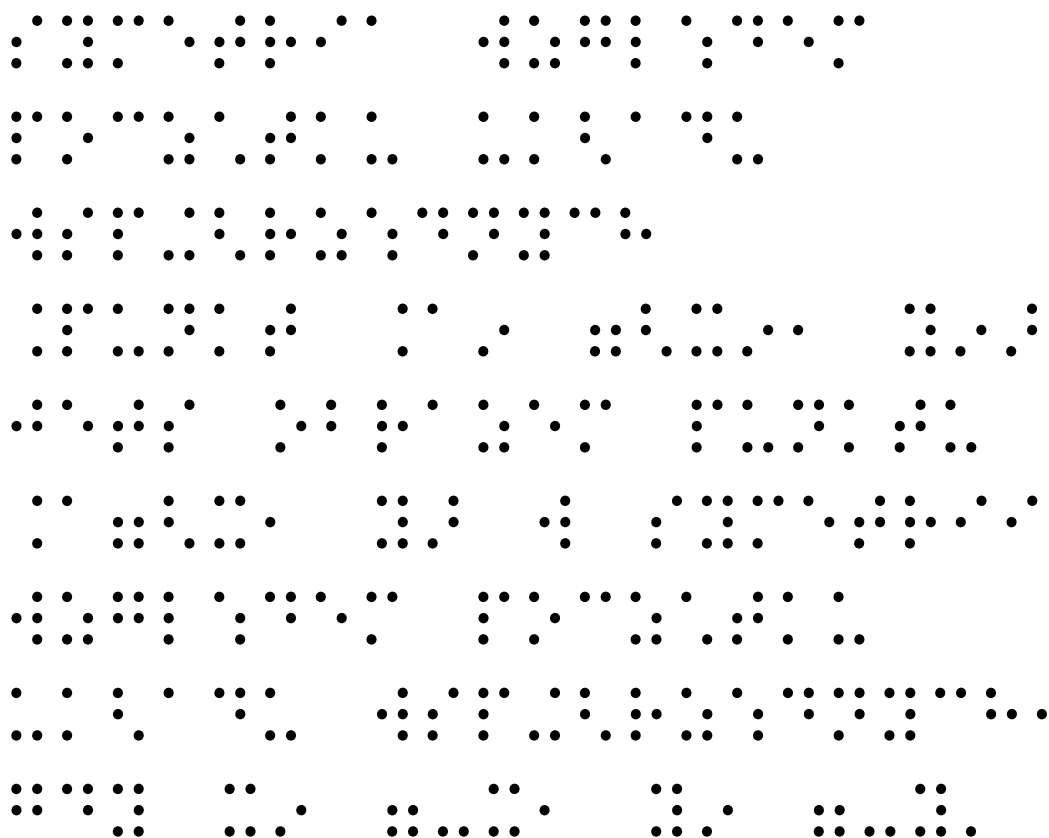




Przekształcenia geometryczne

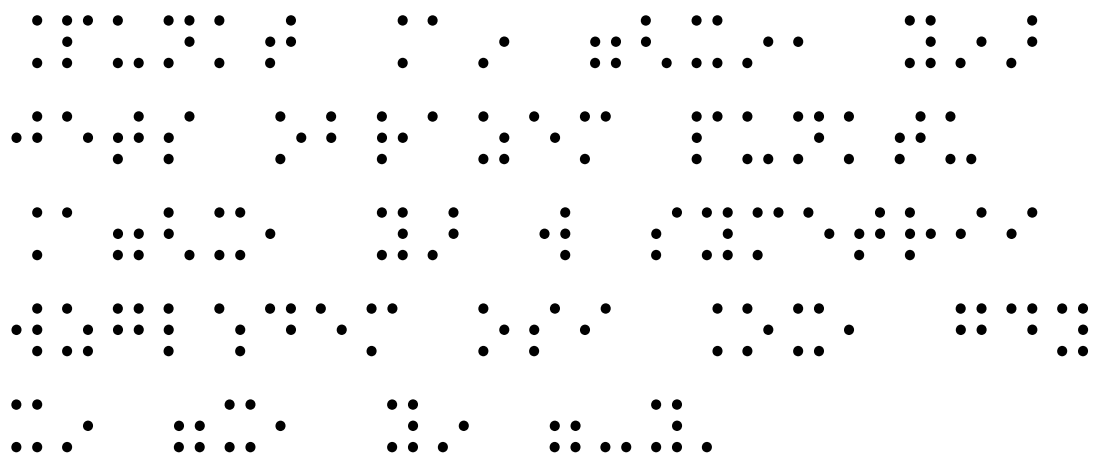


Symetria względem początku układu współrzędnych

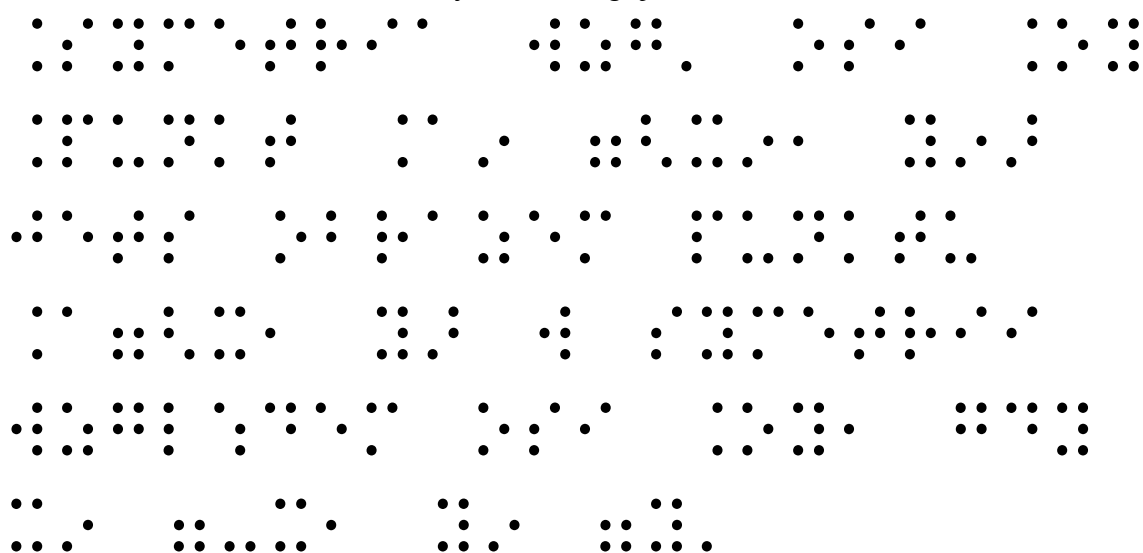


Symetria względem osi OX

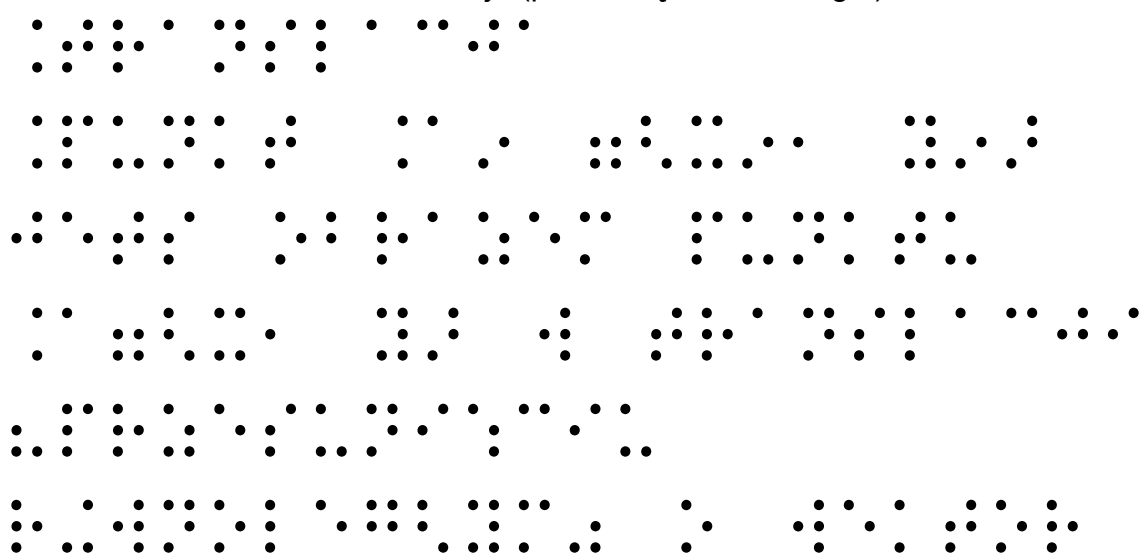




Symetria względem osi OY



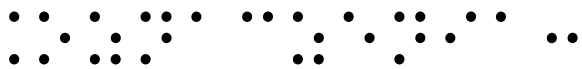
Translacja (przesunięcie równoległe)



A 10x10 grid of 100 small clusters of black dots. Each cluster is a unique arrangement of dots, creating a variety of abstract shapes and patterns. The dots are small and black, set against a white background. The patterns range from simple horizontal or vertical lines to more complex, interconnected shapes that resemble letters or organic forms. The overall effect is a dense, textured field of micro-patterns.

A 4x10 grid of 40 small clusters of black dots. Each cluster is a unique arrangement of dots, creating various abstract shapes and patterns. The clusters are arranged in four rows and ten columns. The patterns range from simple vertical or horizontal lines to more complex, interconnected shapes that resemble letters or symbols. The dots are small and black, set against a white background.

A 3x3 grid of dots. The top row has three dots. The middle row has three dots. The bottom row has two dots, with the leftmost position empty.



A 3x10 grid of dots representing a sparse matrix. The dots are arranged in a pattern that suggests a banded structure with some off-diagonal elements.

Twierdzenie sinusów

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$$

Twierdzenie cosinusów

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

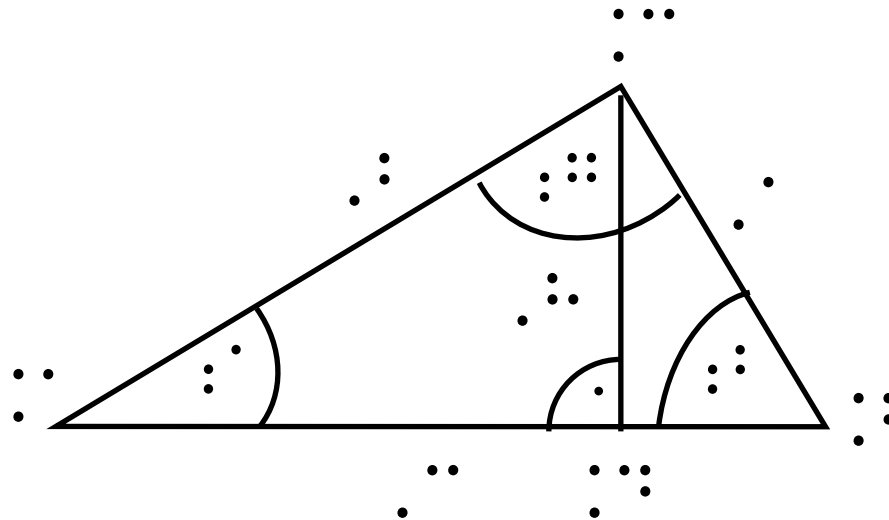
$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Twierdzenie Pitagorasa

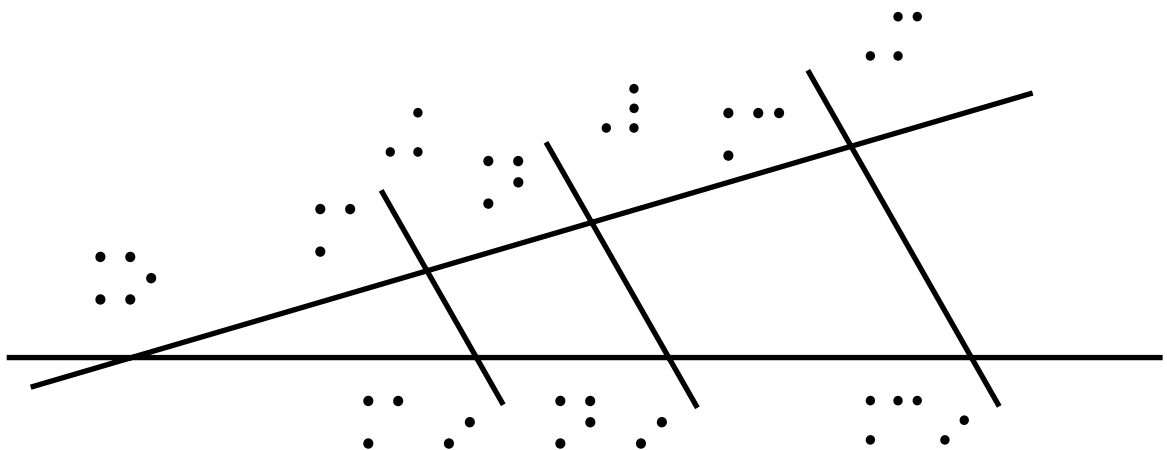
$$a^2 + b^2 = c^2$$

Związki miarowe w trójkącie prostokątnym

$\alpha + \beta = 90^\circ$
 $\alpha + \gamma = 90^\circ$
 $\beta + \gamma = 90^\circ$
 $\alpha + \beta + \gamma = 180^\circ$



$\alpha + \beta = 90^\circ$
 $\alpha + \gamma = 90^\circ$
 $\beta + \gamma = 90^\circ$
 $\alpha + \beta + \gamma = 180^\circ$
 $\alpha + \beta = 90^\circ$
 $\alpha + \gamma = 90^\circ$
 $\beta + \gamma = 90^\circ$
 $\alpha + \beta + \gamma = 180^\circ$
 $\alpha + \beta = 90^\circ$
 $\alpha + \gamma = 90^\circ$
 $\beta + \gamma = 90^\circ$
 $\alpha + \beta + \gamma = 180^\circ$

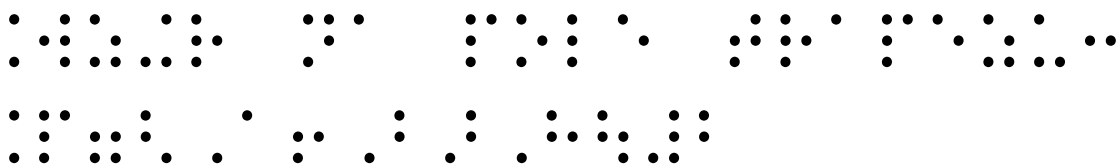
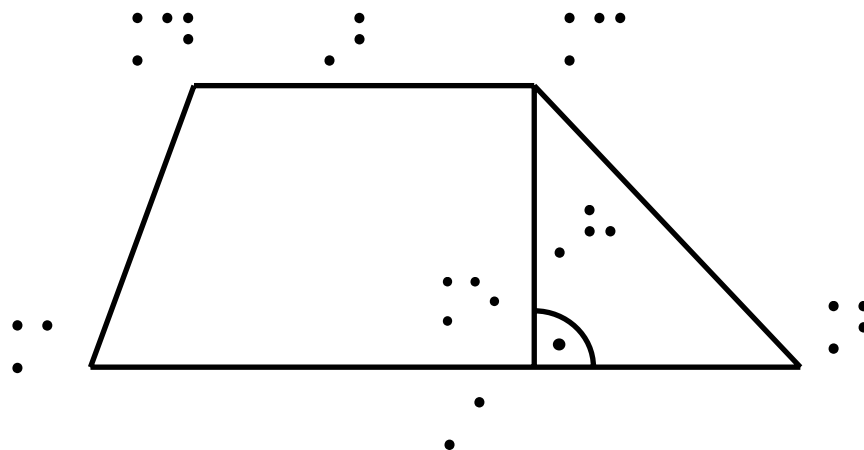
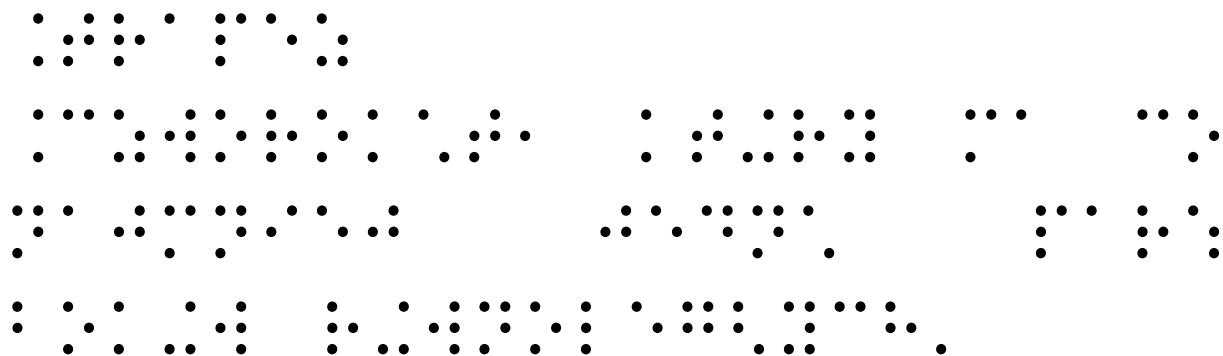


The image displays a 10x10 grid of 100 small dot patterns. Each pattern is a unique combination of the 10 digits (0-9), represented by a specific arrangement of dots. The patterns are arranged in a way that they can be read as a sequence of digits across the rows and columns. The patterns are arranged in a way that they can be read as a sequence of digits across the rows and columns.

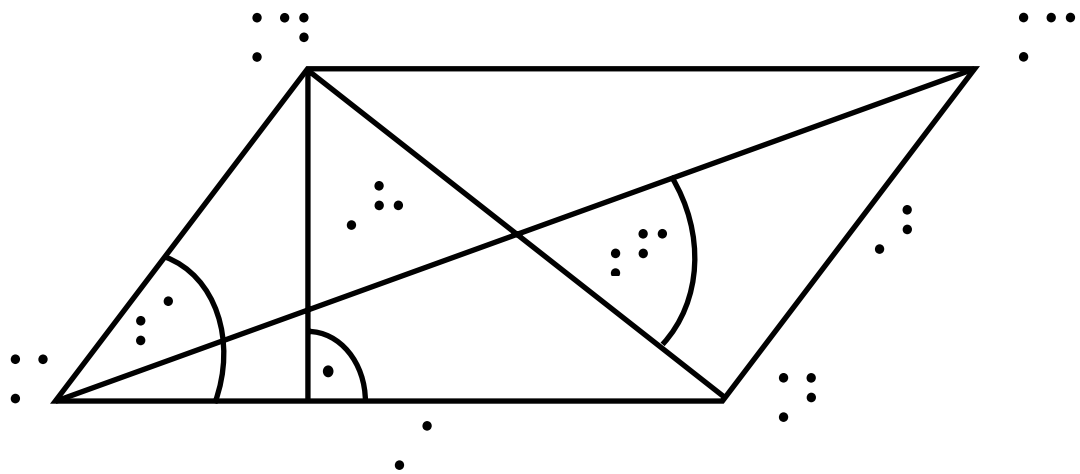
Czworokąty



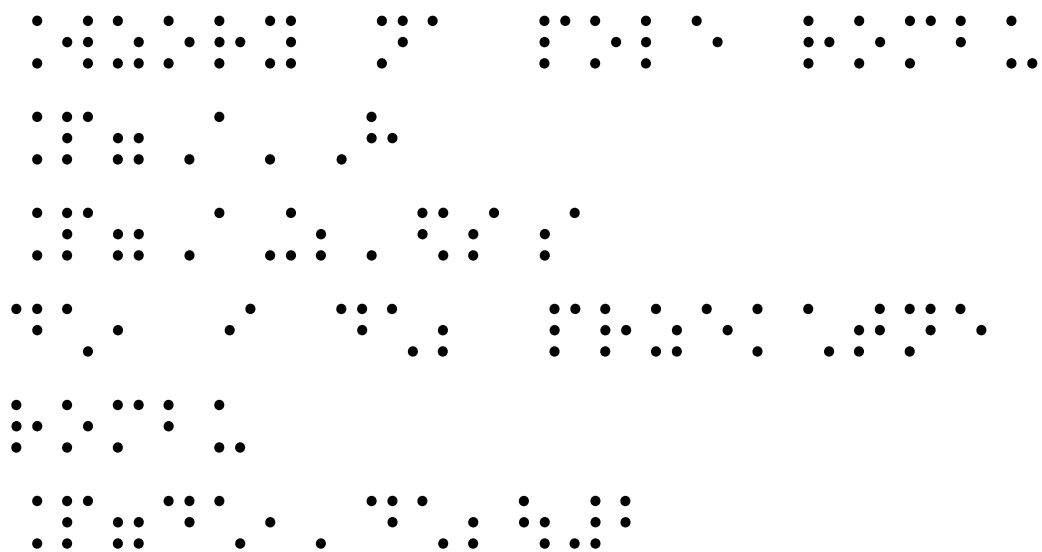
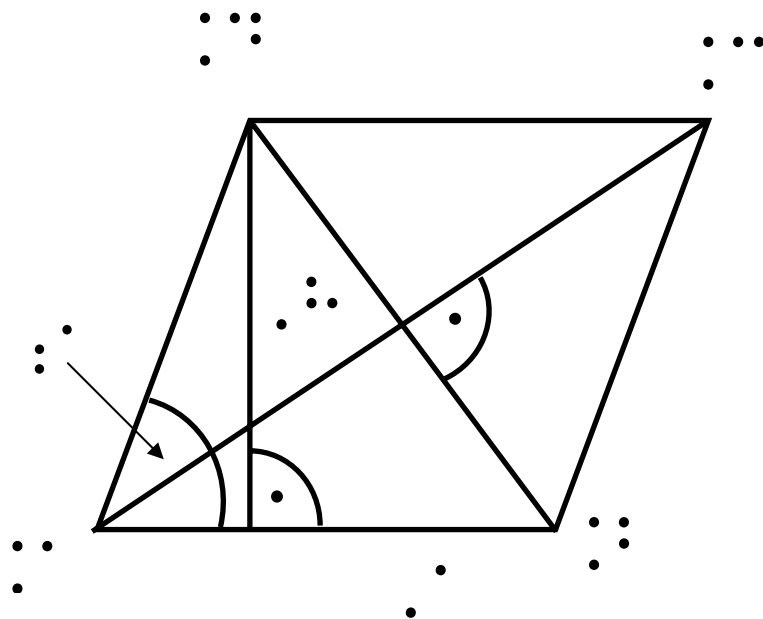
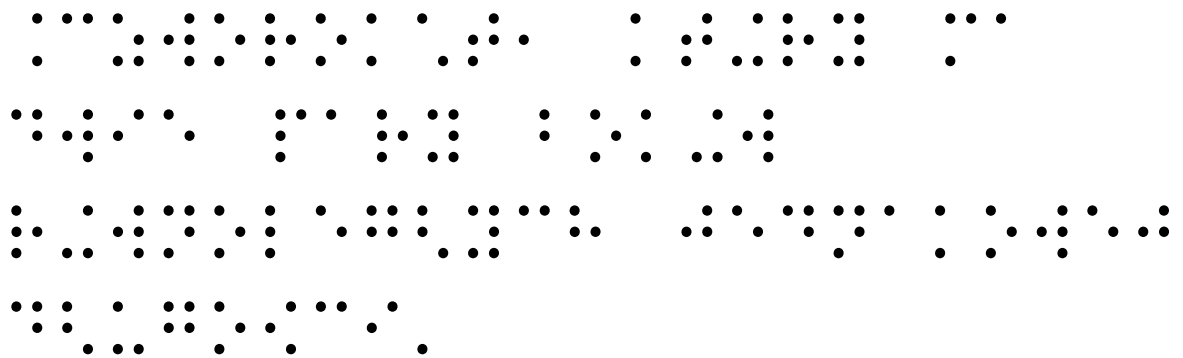
Trapez



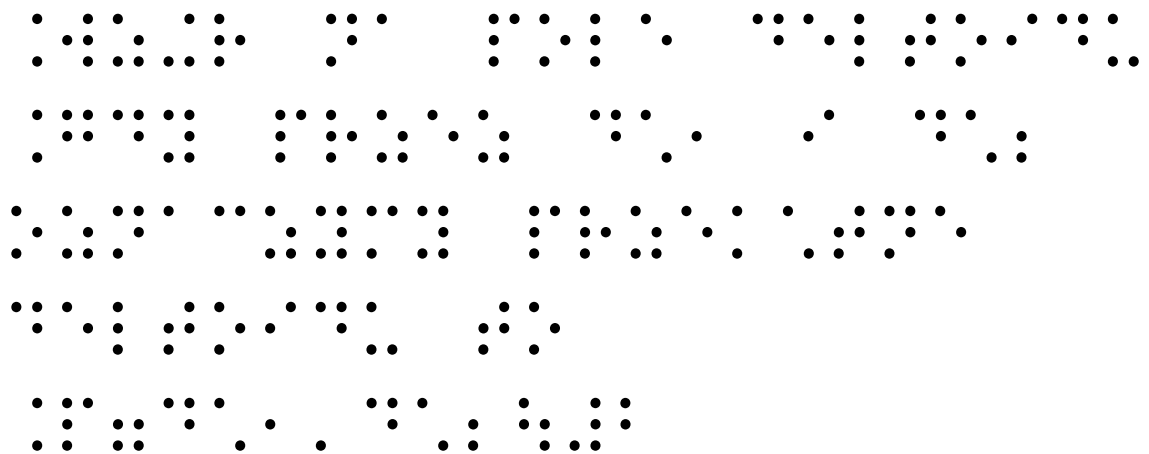
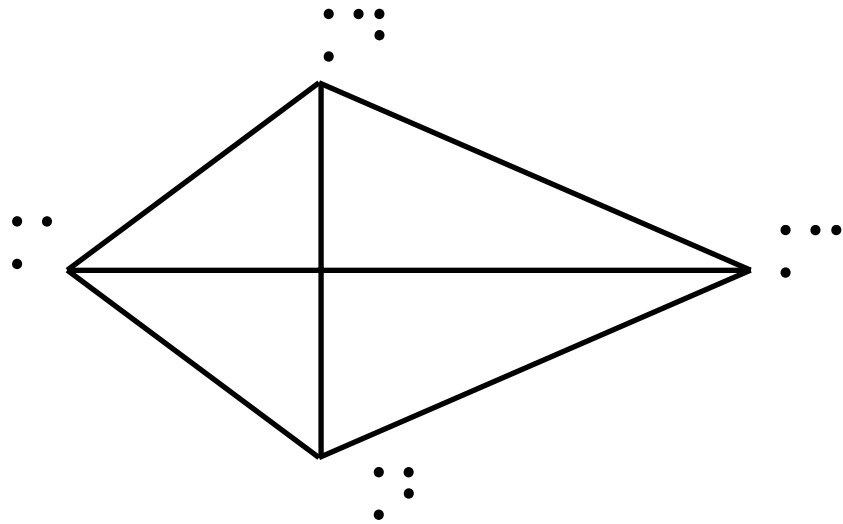
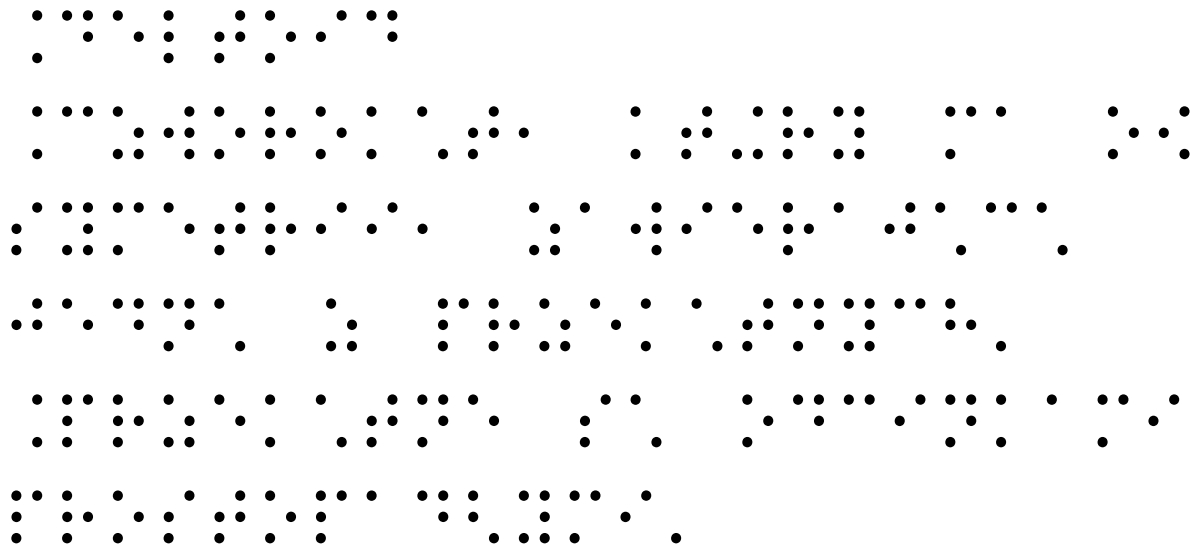
A 4x10 grid of 40 small clusters of black dots. Each cluster represents a different pattern of dot arrangement, likely related to the 20 patterns shown in the previous figure. The patterns vary in the number of dots (from 2 to 10) and their spatial configuration within a small square area.

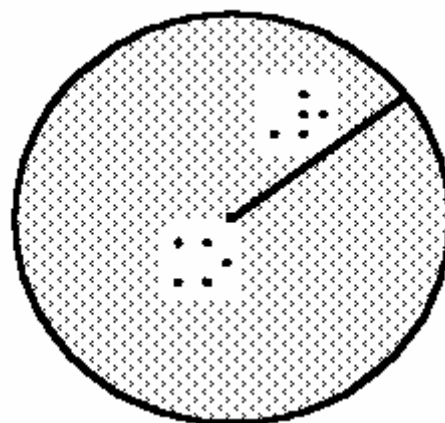


Romb

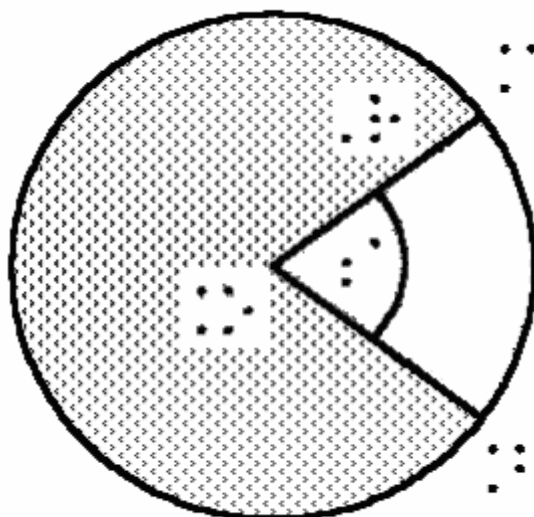


Deltoid





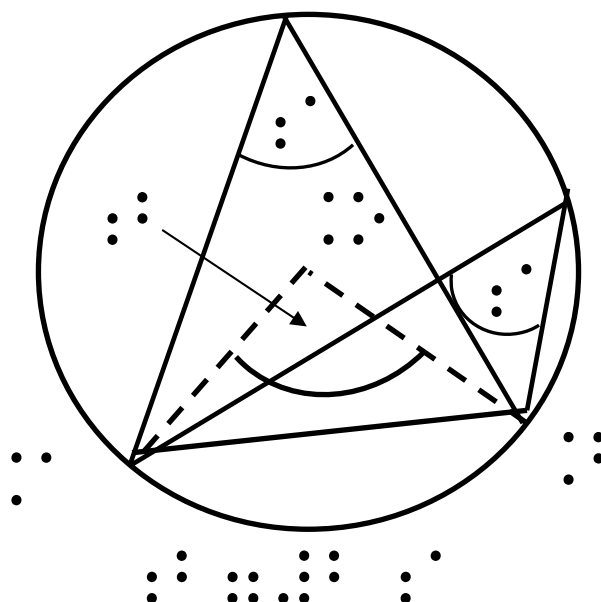
The image shows a 10x10 grid of black dots on a white background. The dots are arranged in a pattern that suggests a banded structure, with some off-diagonal elements. The dots are arranged in a pattern that suggests a banded structure, with some off-diagonal elements. The dots are arranged in a pattern that suggests a banded structure, with some off-diagonal elements.



• • • •

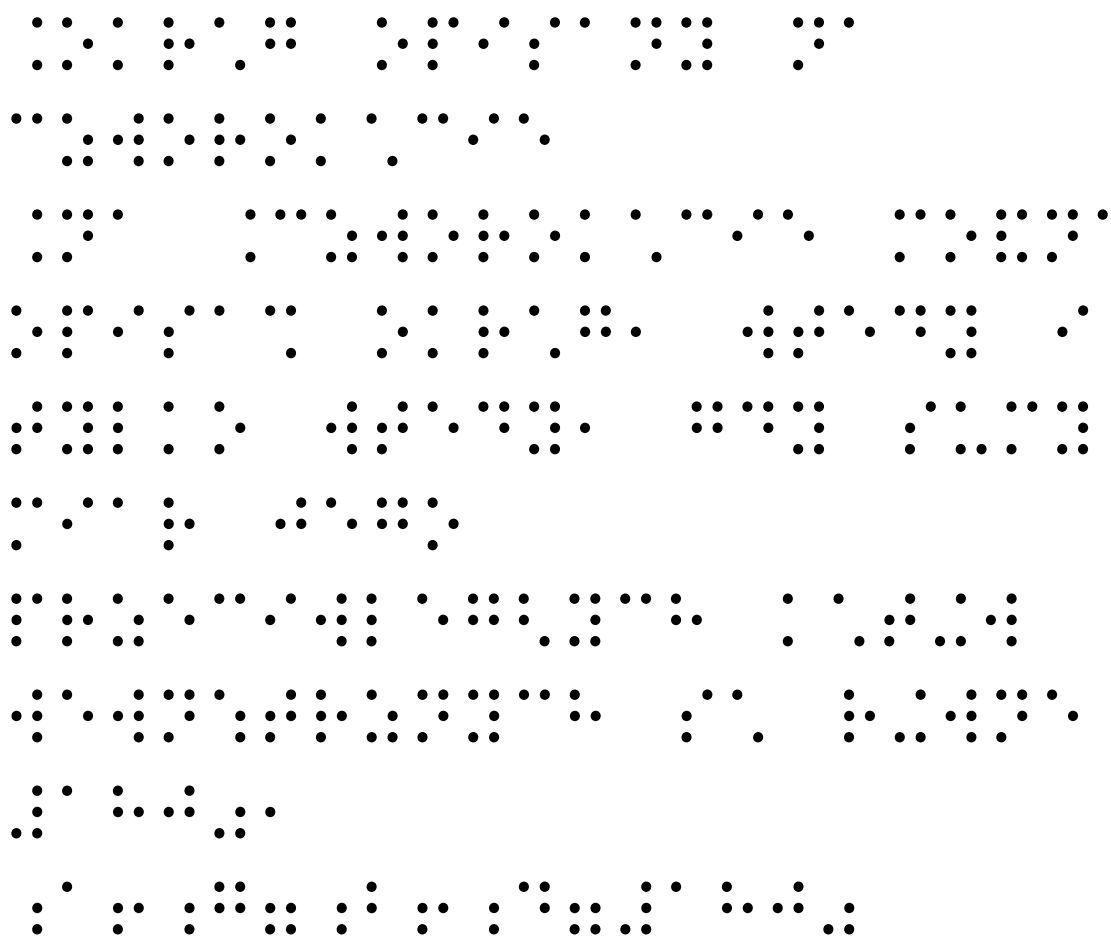
Kąty w okręgu

Kąt wpisany jest kąt, którego wierzchołki i końce boków leżą na okręgu.
 Kąt wpisany jest równy połowie kąta środkowego opierającego się na tym samym łuku.
 Kąt wpisany opierający się na łuku półokręgu jest kątem prostym.
 Kąt wpisany opierający się na łuku mniejszym niż półokrąg jest ostry.
 Kąt wpisany opierający się na łuku większym niż półokrąg jest rozwarty.
 Kąt wpisany opierający się na łuku półokręgu jest kątem prostym.

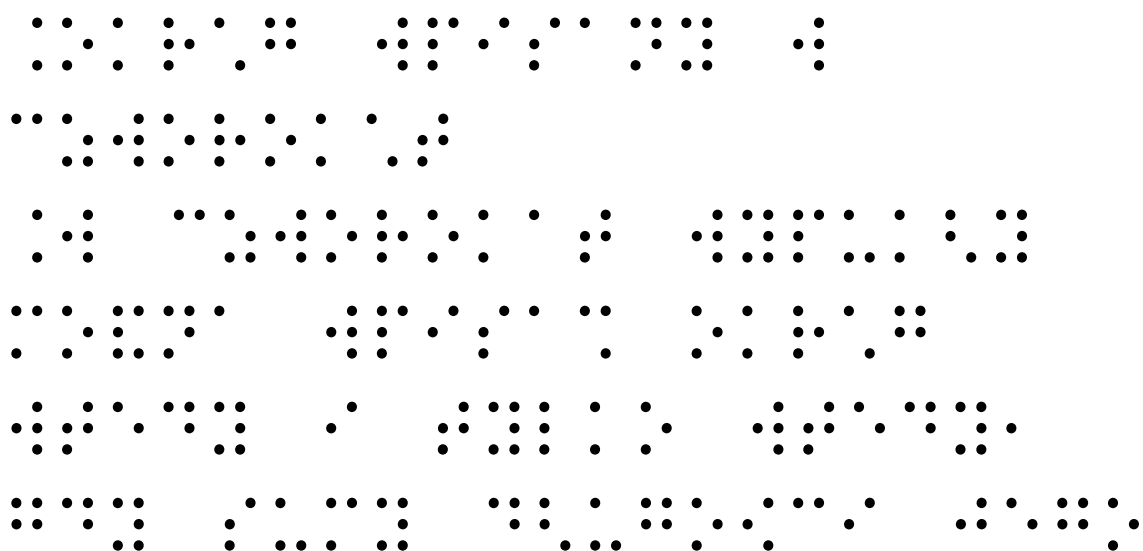


Kąt wpisany jest kąt, którego wierzchołki i końce boków leżą na okręgu.
 Kąt wpisany jest równy połowie kąta środkowego opierającego się na tym samym łuku.
 Kąt wpisany opierający się na łuku półokręgu jest kątem prostym.
 Kąt wpisany opierający się na łuku mniejszym niż półokrąg jest ostry.
 Kąt wpisany opierający się na łuku większym niż półokrąg jest rozwarty.

Okrąg opisany na czworokącie



Okrąg wpisany w czworokąt



1. Wzrost człowieka zmienia się wraz z wiekiem.
 2. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 3. Ciężar ciała człowieka zmienia się wraz z wiekiem.

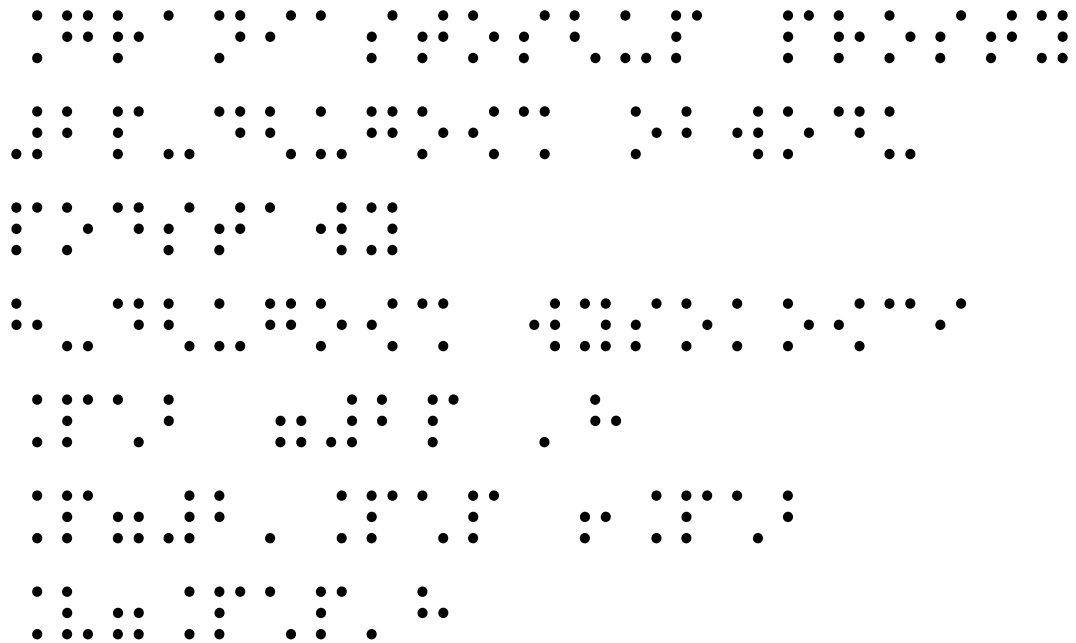
Stereometria

1. Wzrost człowieka zmienia się wraz z wiekiem.
 2. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 3. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 4. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 5. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 6. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 7. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 8. Ciężar ciała człowieka zmienia się wraz z wiekiem.

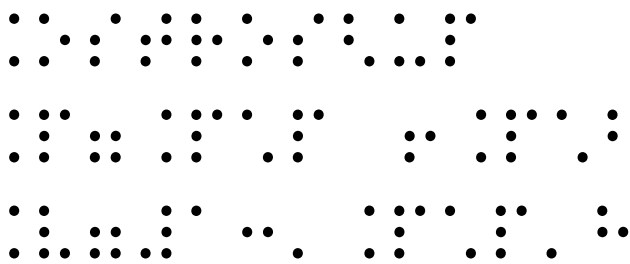
Prostopadłościan

1. Wzrost człowieka zmienia się wraz z wiekiem.
 2. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 3. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 4. Ciężar ciała człowieka zmienia się wraz z wiekiem.
 5. Ciężar ciała człowieka zmienia się wraz z wiekiem.

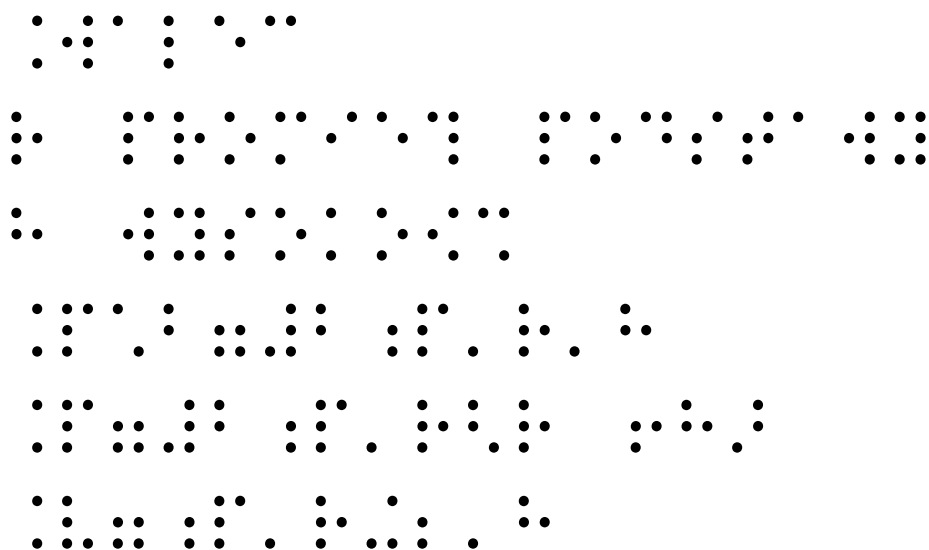
Graniastosłup prosty



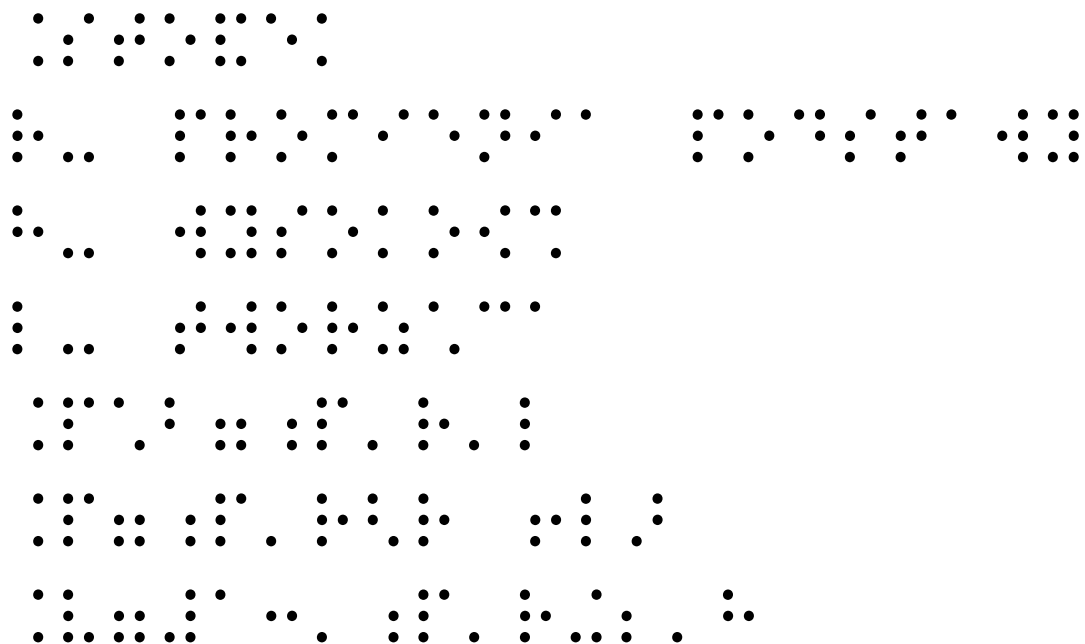
Ostrosłup



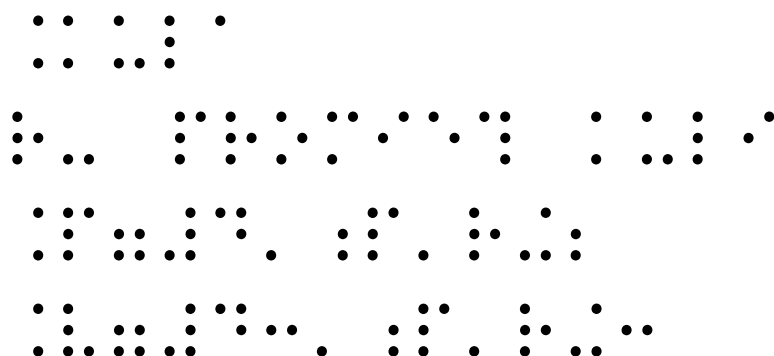
Walec



Stożek



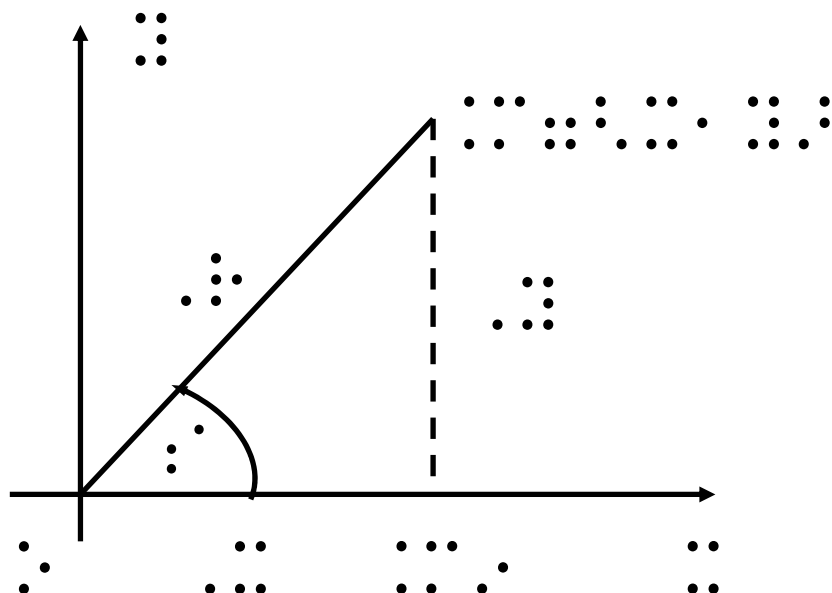
Kula



Trygonometria

Definicje funkcji trygonometrycznych

W trójkącie prostokątnym o kącie α przy wierzchołku O mamy:



Definicje funkcji trygonometrycznych:

- $\sin \alpha = \frac{\text{przyprostokątna przysiadła do kąta}}{\text{przeciętna}}$
- $\cos \alpha = \frac{\text{przyprostokątna przyległa do kąta}}{\text{przeciętna}}$
- $\tan \alpha = \frac{\text{przyprostokątna przysiadła do kąta}}{\text{przyprostokątna przyległa do kąta}}$
- $\cot \alpha = \frac{\text{przyprostokątna przyległa do kąta}}{\text{przyprostokątna przysiadła do kąta}}$
- $\sec \alpha = \frac{\text{przeciętna}}{\text{przyprostokątna przyległa do kąta}}$
- $\csc \alpha = \frac{\text{przeciętna}}{\text{przyprostokątna przysiadła do kąta}}$

Wzajemne zależności:

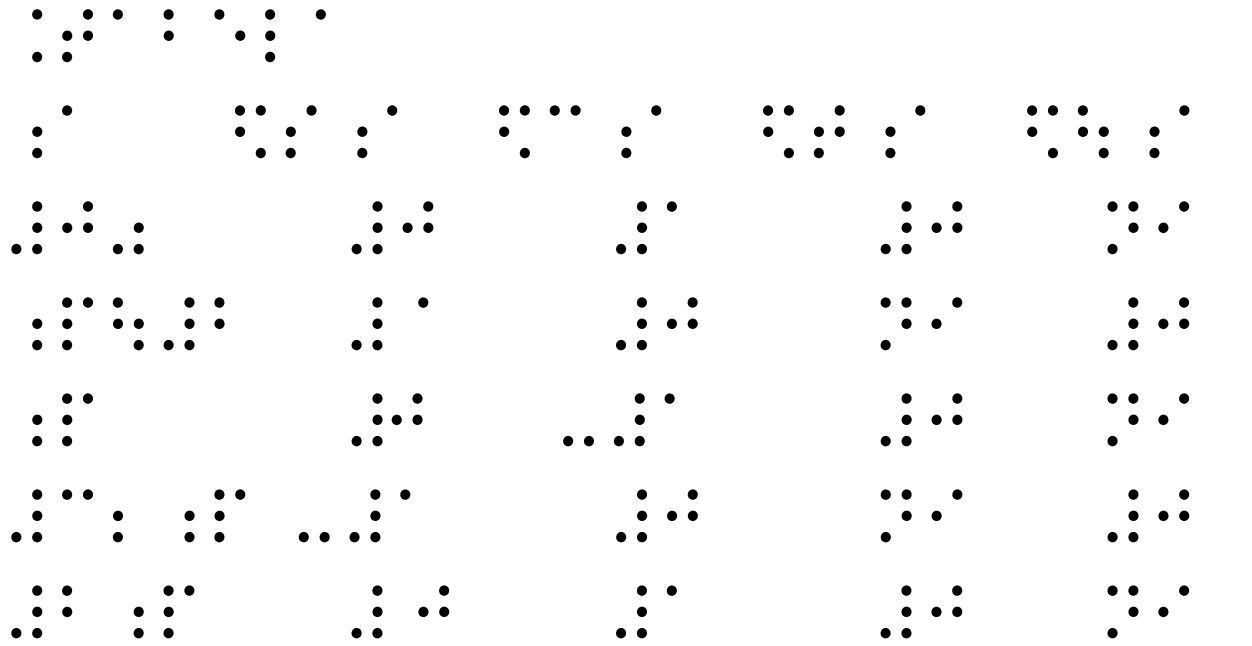
$\sin \alpha = \cos(90^\circ - \alpha)$

$\cos \alpha = \sin(90^\circ - \alpha)$

$\tan \alpha = \cot(90^\circ - \alpha)$

Wartości funkcji trygonometrycznych , gdy $\alpha \in \langle 0, 2\pi \rangle$

$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\csc \alpha$
$\sin 0 = 0$	$\cos 0 = 1$	$\tan 0 = 0$	$\cot 0 = \infty$	$\sec 0 = 1$	$\csc 0 = \infty$
$\sin \frac{\pi}{6} = \frac{1}{2}$	$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$	$\cot \frac{\pi}{6} = \sqrt{3}$	$\sec \frac{\pi}{6} = \frac{2}{\sqrt{3}}$	$\csc \frac{\pi}{6} = 2$
$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\tan \frac{\pi}{4} = 1$	$\cot \frac{\pi}{4} = 1$	$\sec \frac{\pi}{4} = \sqrt{2}$	$\csc \frac{\pi}{4} = \sqrt{2}$
$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos \frac{\pi}{3} = \frac{1}{2}$	$\tan \frac{\pi}{3} = \sqrt{3}$	$\cot \frac{\pi}{3} = \frac{1}{\sqrt{3}}$	$\sec \frac{\pi}{3} = 2$	$\csc \frac{\pi}{3} = \frac{2}{\sqrt{3}}$
$\sin \frac{\pi}{2} = 1$	$\cos \frac{\pi}{2} = 0$	$\tan \frac{\pi}{2} = \infty$	$\cot \frac{\pi}{2} = 0$	$\sec \frac{\pi}{2} = \infty$	$\csc \frac{\pi}{2} = 1$
$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos \frac{2\pi}{3} = -\frac{1}{2}$	$\tan \frac{2\pi}{3} = -\sqrt{3}$	$\cot \frac{2\pi}{3} = -\frac{1}{\sqrt{3}}$	$\sec \frac{2\pi}{3} = -2$	$\csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}}$
$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$	$\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$	$\tan \frac{3\pi}{4} = -1$	$\cot \frac{3\pi}{4} = -1$	$\sec \frac{3\pi}{4} = -\sqrt{2}$	$\csc \frac{3\pi}{4} = \sqrt{2}$
$\sin \frac{5\pi}{6} = \frac{1}{2}$	$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$	$\tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$	$\cot \frac{5\pi}{6} = -\sqrt{3}$	$\sec \frac{5\pi}{6} = -\frac{2}{\sqrt{3}}$	$\csc \frac{5\pi}{6} = 2$
$\sin \pi = 0$	$\cos \pi = -1$	$\tan \pi = 0$	$\cot \pi = \infty$	$\sec \pi = -1$	$\csc \pi = \infty$
$\sin \frac{7\pi}{6} = -\frac{1}{2}$	$\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$	$\tan \frac{7\pi}{6} = \frac{1}{\sqrt{3}}$	$\cot \frac{7\pi}{6} = \sqrt{3}$	$\sec \frac{7\pi}{6} = -\frac{2}{\sqrt{3}}$	$\csc \frac{7\pi}{6} = -2$
$\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$	$\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$	$\tan \frac{5\pi}{4} = 1$	$\cot \frac{5\pi}{4} = 1$	$\sec \frac{5\pi}{4} = -\sqrt{2}$	$\csc \frac{5\pi}{4} = -\sqrt{2}$
$\sin \frac{3\pi}{2} = -1$	$\cos \frac{3\pi}{2} = 0$	$\tan \frac{3\pi}{2} = \infty$	$\cot \frac{3\pi}{2} = 0$	$\sec \frac{3\pi}{2} = \infty$	$\csc \frac{3\pi}{2} = -1$
$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$	$\cos \frac{4\pi}{3} = \frac{1}{2}$	$\tan \frac{4\pi}{3} = -\sqrt{3}$	$\cot \frac{4\pi}{3} = -\frac{1}{\sqrt{3}}$	$\sec \frac{4\pi}{3} = 2$	$\csc \frac{4\pi}{3} = -\frac{2}{\sqrt{3}}$
$\sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$	$\cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$	$\tan \frac{7\pi}{4} = -1$	$\cot \frac{7\pi}{4} = -1$	$\sec \frac{7\pi}{4} = \sqrt{2}$	$\csc \frac{7\pi}{4} = -\sqrt{2}$
$\sin \frac{11\pi}{6} = -\frac{1}{2}$	$\cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan \frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$	$\cot \frac{11\pi}{6} = -\sqrt{3}$	$\sec \frac{11\pi}{6} = \frac{2}{\sqrt{3}}$	$\csc \frac{11\pi}{6} = -2$



Związki między funkcjami trygonometrycznymi

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \sin^2 x &= 1 - \cos^2 x \\ \cos^2 x &= 1 - \sin^2 x \\ \sin^2 x &= \frac{1 - \cos 2x}{2} \\ \cos^2 x &= \frac{1 + \cos 2x}{2} \\ \sin^2 x &= \frac{1 - \cos 2x}{2} \\ \cos^2 x &= \frac{1 + \cos 2x}{2} \\ \sin^2 x &= \frac{1 - \cos 2x}{2} \\ \cos^2 x &= \frac{1 + \cos 2x}{2} \\ \sin^2 x &= \frac{1 - \cos 2x}{2} \\ \cos^2 x &= \frac{1 + \cos 2x}{2} \end{aligned}$$

Wartości funkcji trygonometrycznych (30° , 45° , 60°)

$\sin 30^\circ$	$\sin 45^\circ$	$\sin 60^\circ$	$\cos 30^\circ$	$\cos 45^\circ$	$\cos 60^\circ$
$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

$\tan 30^\circ$	$\tan 45^\circ$	$\tan 60^\circ$	$\cot 30^\circ$	$\cot 45^\circ$	$\cot 60^\circ$
$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$
$\sec 30^\circ$	$\sec 45^\circ$	$\sec 60^\circ$	$\csc 30^\circ$	$\csc 45^\circ$	$\csc 60^\circ$
$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$
$\sec 30^\circ$	$\sec 45^\circ$	$\sec 60^\circ$	$\csc 30^\circ$	$\csc 45^\circ$	$\csc 60^\circ$
$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$

Wzory redukcyjne

$\sin(\alpha + 2\pi) = \sin \alpha$	$\sin(\alpha + \pi) = -\sin \alpha$	$\sin(\alpha - \pi) = -\sin \alpha$	$\sin(\alpha - 2\pi) = \sin \alpha$
$\cos(\alpha + 2\pi) = \cos \alpha$	$\cos(\alpha + \pi) = -\cos \alpha$	$\cos(\alpha - \pi) = -\cos \alpha$	$\cos(\alpha - 2\pi) = \cos \alpha$
$\tan(\alpha + \pi) = \tan \alpha$	$\tan(\alpha - \pi) = \tan \alpha$	$\cot(\alpha + \pi) = \cot \alpha$	$\cot(\alpha - \pi) = \cot \alpha$
$\sec(\alpha + 2\pi) = \sec \alpha$	$\sec(\alpha - 2\pi) = \sec \alpha$	$\csc(\alpha + 2\pi) = \csc \alpha$	$\csc(\alpha - 2\pi) = \csc \alpha$
$\sin(\pi - \alpha) = \sin \alpha$	$\sin(\pi + \alpha) = -\sin \alpha$	$\sin(-\alpha) = -\sin \alpha$	$\sin(\alpha) = \sin(\pi - \alpha)$
$\cos(\pi - \alpha) = -\cos \alpha$	$\cos(\pi + \alpha) = -\cos \alpha$	$\cos(-\alpha) = \cos \alpha$	$\cos(\alpha) = \cos(\pi - \alpha)$
$\tan(\pi - \alpha) = -\tan \alpha$	$\tan(\pi + \alpha) = \tan \alpha$	$\tan(-\alpha) = -\tan \alpha$	$\tan(\alpha) = -\tan(\pi - \alpha)$
$\cot(\pi - \alpha) = -\cot \alpha$	$\cot(\pi + \alpha) = \cot \alpha$	$\cot(-\alpha) = -\cot \alpha$	$\cot(\alpha) = -\cot(\pi - \alpha)$



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Funkcje podwojonego kąta

$\sin 2\alpha = 2 \sin \alpha \cos \alpha$
 $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
 $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$
 $\cot 2\alpha = \frac{1 - \tan^2 \alpha}{2 \tan \alpha}$
 $\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$
 $\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$
 $\tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$
 $\cot^2 \alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$
 $\sin \alpha \cos \alpha = \frac{\sin 2\alpha}{2}$
 $\cos \alpha \sin \alpha = \frac{\sin 2\alpha}{2}$
 $\cos \alpha \cos \alpha = \frac{1 + \cos 2\alpha}{2}$
 $\sin \alpha \sin \alpha = \frac{1 - \cos 2\alpha}{2}$
 $\cos \alpha \tan \alpha = \sin \alpha$
 $\sin \alpha \tan \alpha = \sin^2 \alpha$
 $\cos \alpha \cot \alpha = \cos^2 \alpha$
 $\sin \alpha \cot \alpha = \sin \alpha \cos \alpha$
 $\cos \alpha \sec \alpha = \sec \alpha$
 $\sin \alpha \sec \alpha = \tan \alpha$
 $\cos \alpha \csc \alpha = \csc \alpha$
 $\sin \alpha \csc \alpha = 1$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

Funkcje kąta potrojonego

$$\sin 3\alpha = 3\sin \alpha - 4\sin^3 \alpha$$

$$\cos 3\alpha = 4\cos^3 \alpha - 3\cos \alpha$$

$$\sin \frac{\alpha}{3} = \frac{1}{4} \sqrt{3 - 4\cos \alpha}$$

$$\cos \frac{\alpha}{3} = \frac{1}{4} \sqrt{4\cos \alpha + 3}$$

$$\sin \frac{\alpha}{2} = \frac{1}{2} \sqrt{2 - 2\cos \alpha}$$

$$\cos \frac{\alpha}{2} = \frac{1}{2} \sqrt{2 + 2\cos \alpha}$$

$$\sin \frac{\alpha}{4} = \frac{1}{4} \sqrt{2 - 2\cos \frac{\alpha}{2}}$$

$$\cos \frac{\alpha}{4} = \frac{1}{4} \sqrt{2 + 2\cos \frac{\alpha}{2}}$$

Sumy i różnice funkcji trygonometrycznych

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2}$$

$$\cos \alpha \pm \cos \beta = 2 \cos \frac{\alpha \pm \beta}{2} \sin \frac{\alpha \mp \beta}{2}$$

$$\sin \frac{\alpha}{2} = \frac{1}{2} \sqrt{2 - 2\cos \alpha}$$

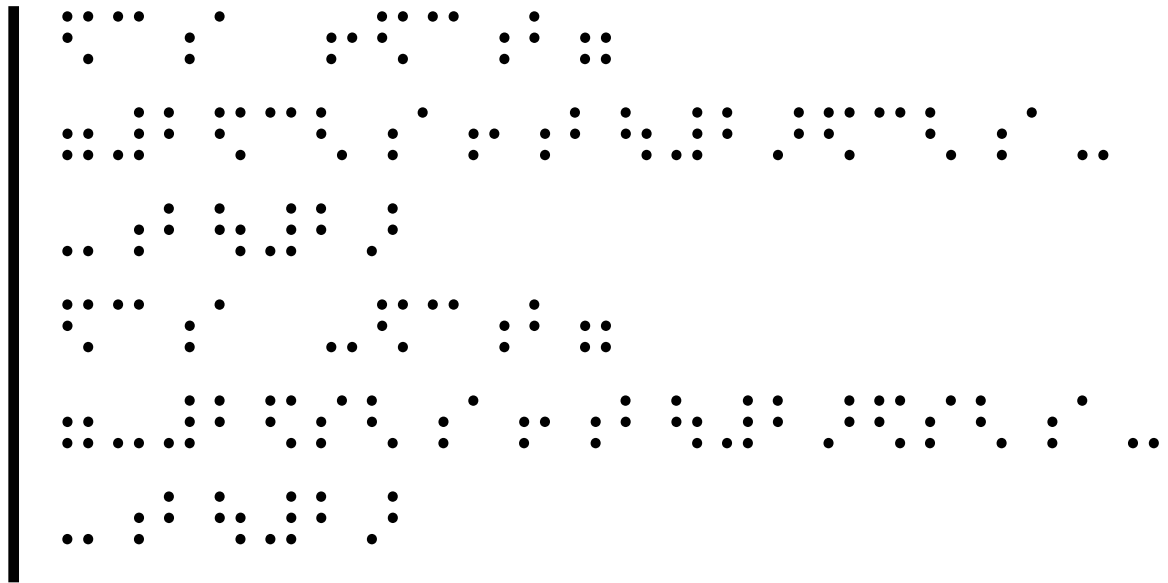
$$\cos \frac{\alpha}{2} = \frac{1}{2} \sqrt{2 + 2\cos \alpha}$$

$$\sin \frac{\alpha}{4} = \frac{1}{4} \sqrt{2 - 2\cos \frac{\alpha}{2}}$$

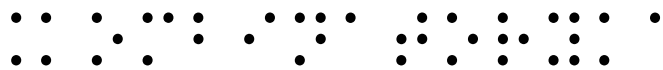
$$\cos \frac{\alpha}{4} = \frac{1}{4} \sqrt{2 + 2\cos \frac{\alpha}{2}}$$

$$\sin \frac{\alpha}{8} = \frac{1}{8} \sqrt{2 - 2\cos \frac{\alpha}{4}}$$

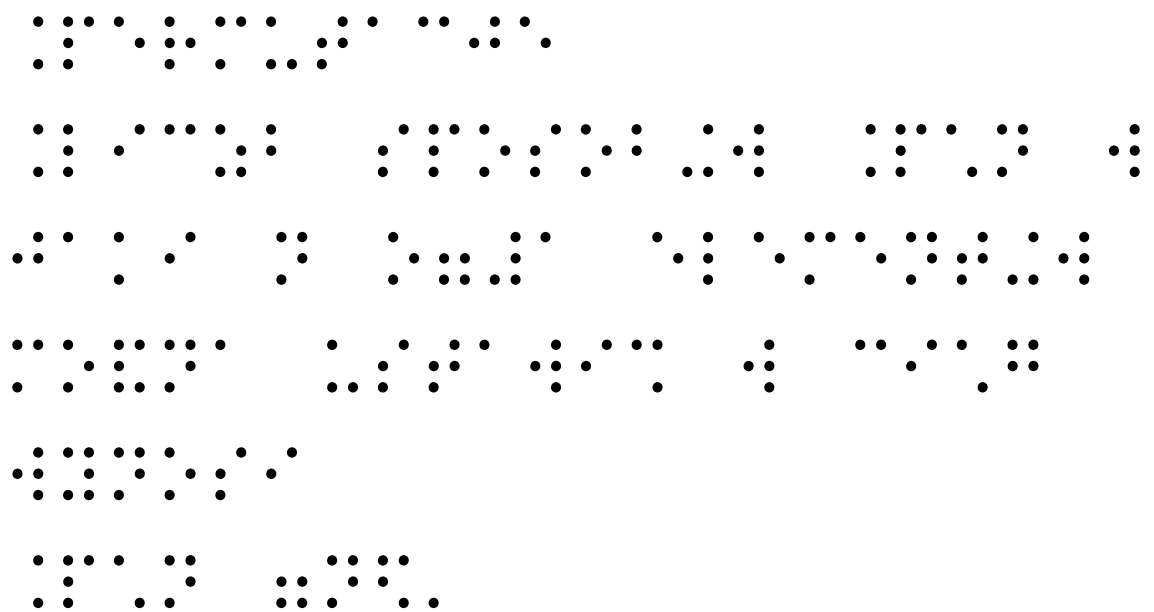
$$\cos \frac{\alpha}{8} = \frac{1}{8} \sqrt{2 + 2\cos \frac{\alpha}{4}}$$



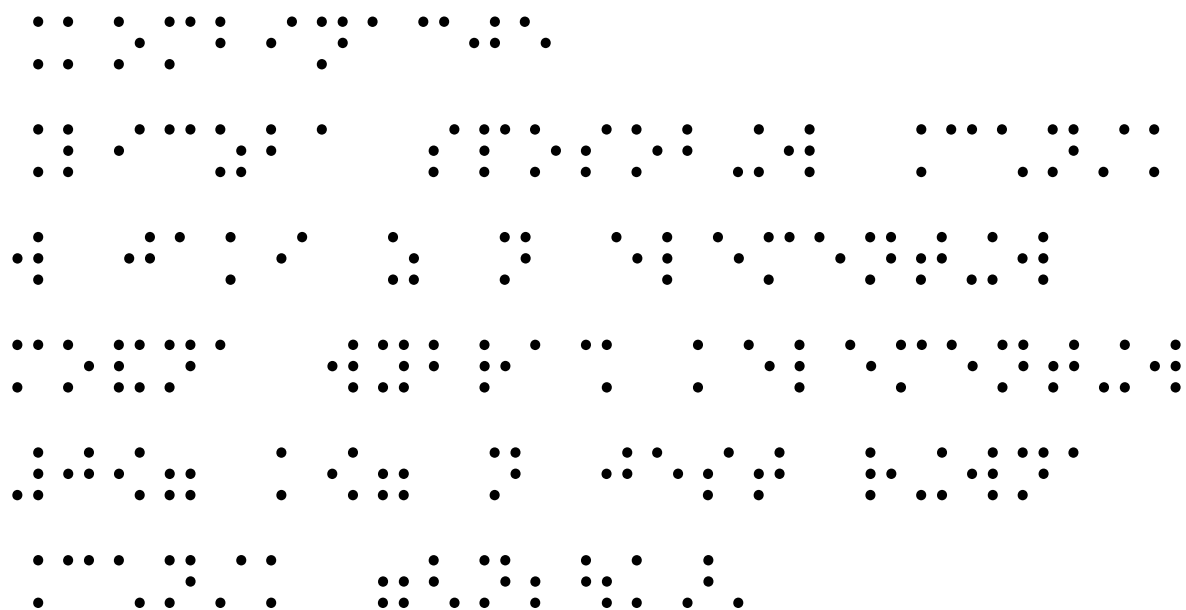
Kombinatoryka



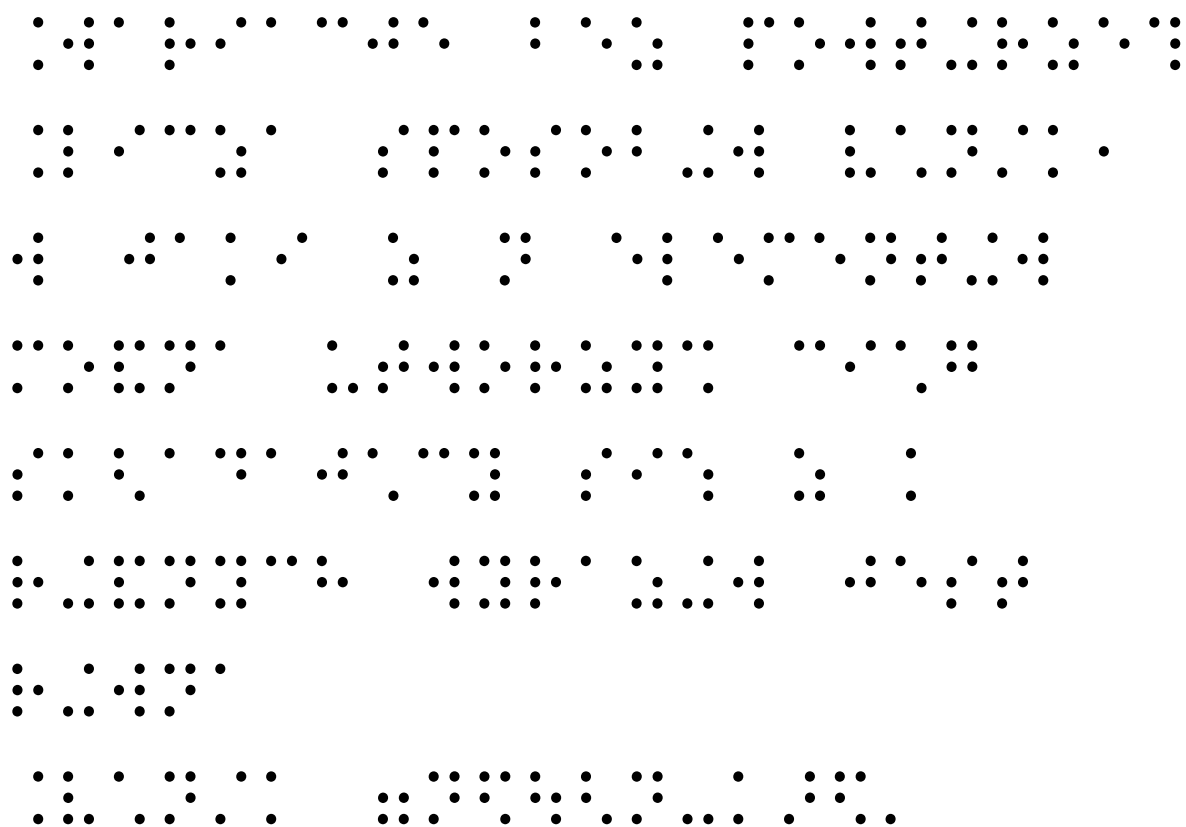
Permutacje



Kombinacje



Wariacje bez powtórzeń



Wariacje z powtórzeniami

$$\frac{n!}{k_1! k_2! \dots k_r!}$$

$$= \frac{n!}{k_1! k_2! \dots k_r!}$$

$$= \frac{n!}{k_1! k_2! \dots k_r!}$$

$$= \frac{n!}{k_1! k_2! \dots k_r!}$$

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$$= \frac{n!}{k_1! k_2! \dots k_r!}$$

$$= \frac{n!}{k_1! k_2! \dots k_r!}$$

Rachunek prawdopodobieństwa

$$P(A) = \frac{|A|}{|\Omega|}$$

$$P(A \cap B) = \frac{|A \cap B|}{|\Omega|}$$

Klasyczna definicja

$$P(A) = \frac{|A|}{|\Omega|}$$

$$P(A \cap B) = \frac{|A \cap B|}{|\Omega|}$$

$$P(A \cup B) = \frac{|A \cup B|}{|\Omega|}$$

$$P(A \setminus B) = \frac{|A \setminus B|}{|\Omega|}$$

A 10x10 grid of black dots on a white background. The dots are arranged in a sparse, abstract pattern, with some rows having more dots than others. The pattern is roughly as follows:
Row 1: 10 dots
Row 2: 10 dots
Row 3: 10 dots
Row 4: 10 dots
Row 5: 10 dots
Row 6: 10 dots
Row 7: 10 dots
Row 8: 10 dots
Row 9: 10 dots
Row 10: 10 dots
The dots are arranged in a way that creates a sense of depth and perspective, with some dots appearing closer than others. The overall effect is a minimalist, geometric composition.

Własności prawdopodobieństwa

A 15x15 grid of black dots on a white background. The dots are arranged in a sparse, abstract pattern that vaguely suggests a stylized face or figure. The pattern is composed of several distinct clusters of dots, some of which form recognizable shapes like eyes, a nose, and a mouth, though the overall image is fragmented and pixelated. The dots are black and the background is white.

$P(A \cap B) = P(A) \cdot P(B)$
 $P(A \cap B) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$
 $P(A \cap B) = \frac{1}{16}$
 $P(A \cap B) = \frac{1}{16}$

Zdarzenia niezależne

$P(A \cap B) = P(A) \cdot P(B)$
 $P(A \cap B) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$
 $P(A \cap B) = \frac{1}{16}$
 $P(A \cap B) = \frac{1}{16}$
 $P(A \cap B) = \frac{1}{16}$
 $P(A \cap B) = \frac{1}{16}$

Prawdopodobieństwo warunkowe

$P(A|B) = \frac{P(A \cap B)}{P(B)}$
 $P(A|B) = \frac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$
 $P(A|B) = \frac{1}{4}$
 $P(A|B) = \frac{1}{4}$
 $P(A|B) = \frac{1}{4}$
 $P(A|B) = \frac{1}{4}$

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Twierdzenie o prawdopodobieństwie całkowitym

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Twierdzenie Bernoullego

$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$
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 $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$

1. Wzrost i waga ciała
 2. Ciężar ciała
 3. Ciężar ciała
 4. Ciężar ciała
 5. Ciężar ciała
 6. Ciężar ciała
 7. Ciężar ciała
 8. Ciężar ciała

Parametry danych statystycznych

1. Wzrost i waga ciała
 2. Ciężar ciała
 3. Ciężar ciała

Średnia arytmetyczna

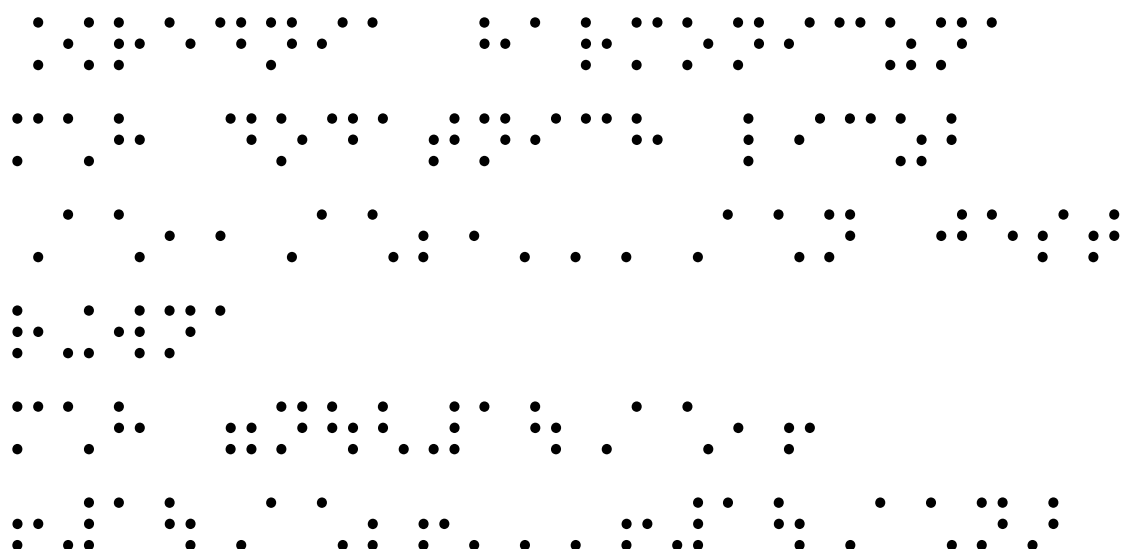
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 6. Ciężar ciała
 7. Ciężar ciała
 8. Ciężar ciała

The image displays a 10x10 grid of black dots on a white background. The dots are arranged in a pattern that suggests a banded structure, with a higher density of dots along the main diagonal and some off-diagonal elements, particularly in the upper and lower triangles. This pattern is characteristic of a sparse matrix, often used in numerical methods for solving differential equations or representing a discretized system. The dots are small and black, and the grid is perfectly square.

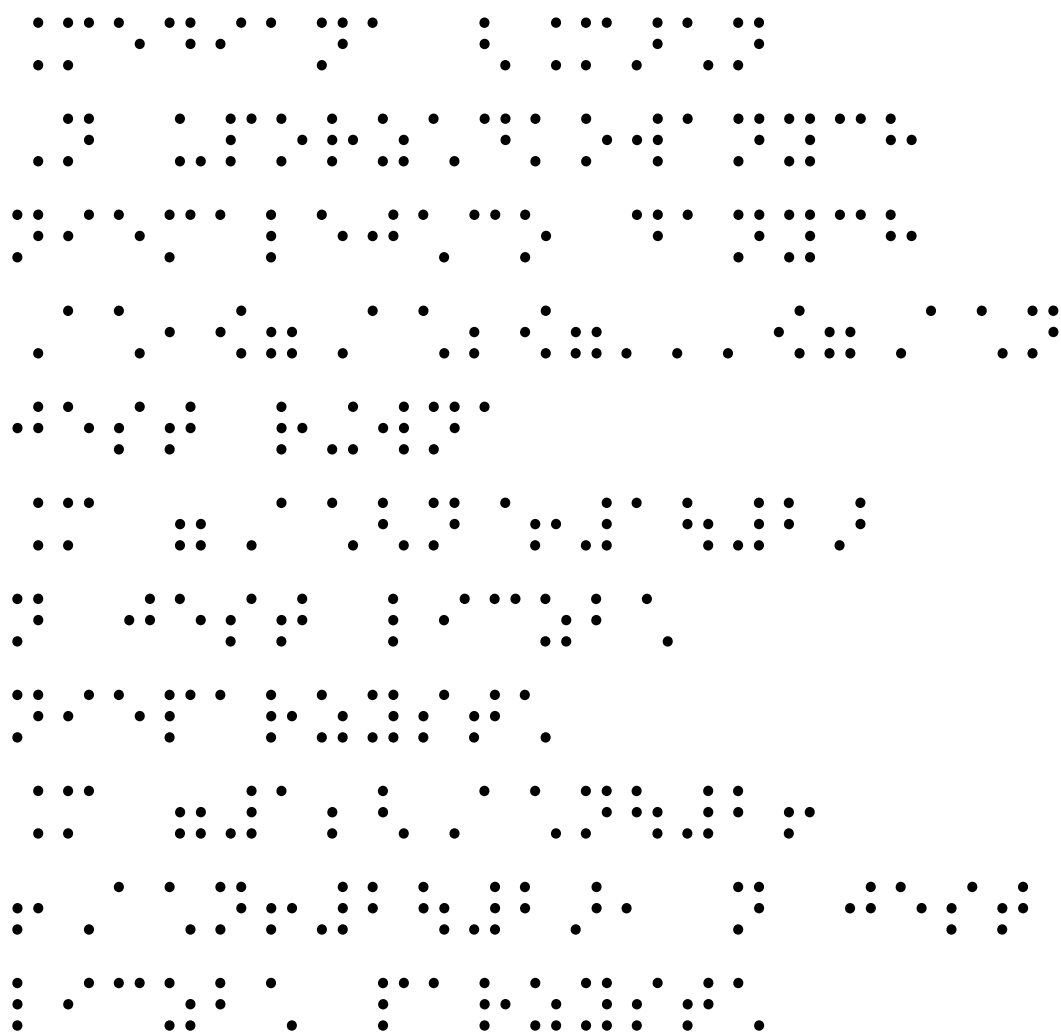
A 10x10 grid of black dots on a white background. The dots are arranged in a sparse, non-uniform pattern, suggesting a network or graph structure. The dots are concentrated in the upper half of the grid, with a few scattered dots in the lower half. The overall shape is roughly rectangular, with some dots forming a central cluster and others forming smaller groups or isolated points.



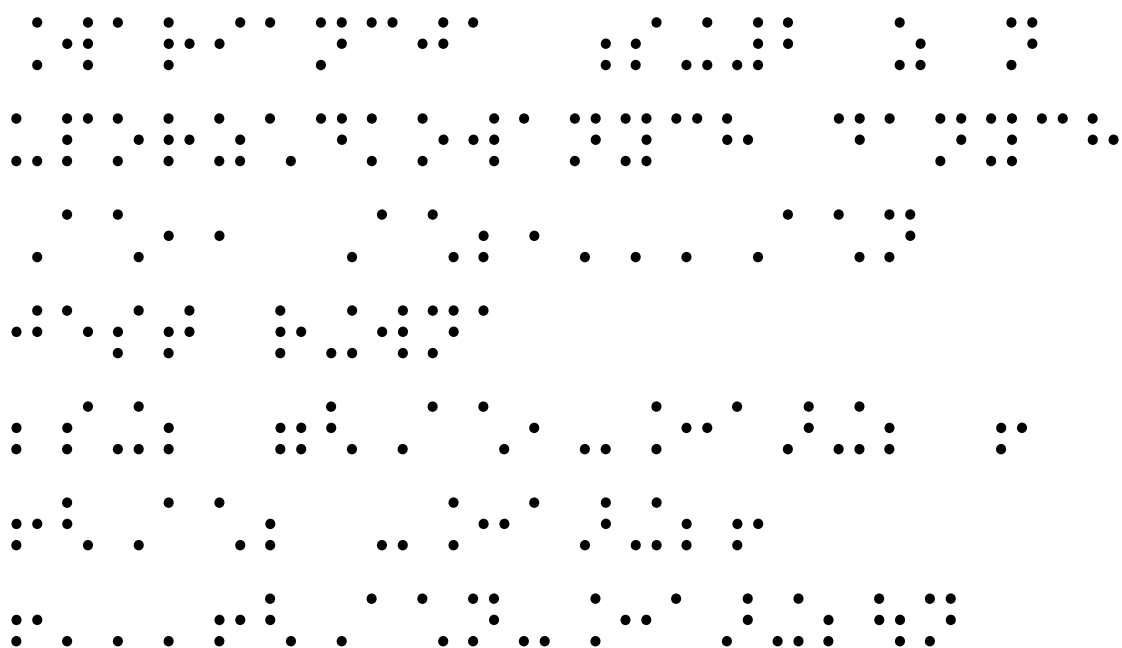
Średnia harmoniczna



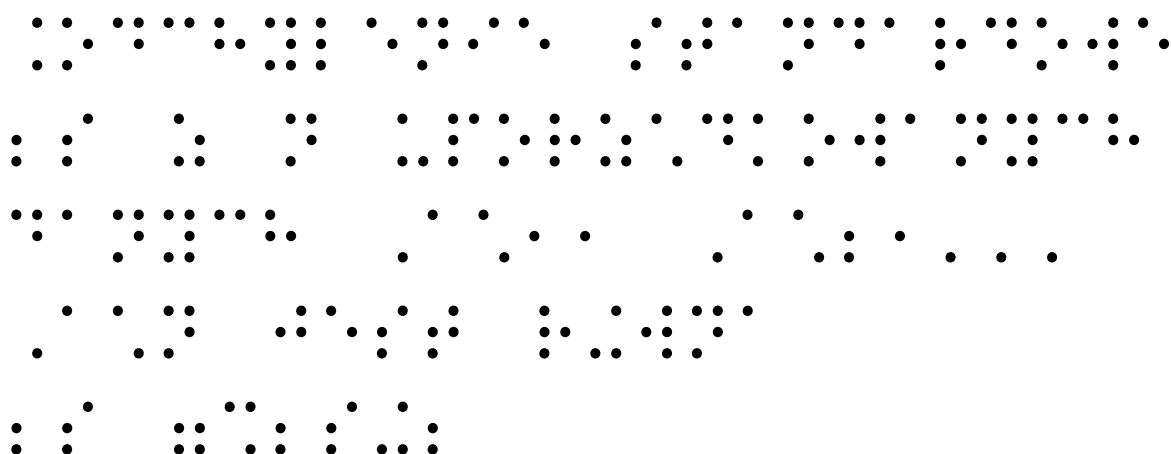
Mediana



Wariancja



Odchylenie standardowe



TABLICA FUNKCJI TRYGONOMETRYCZNYCH

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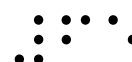
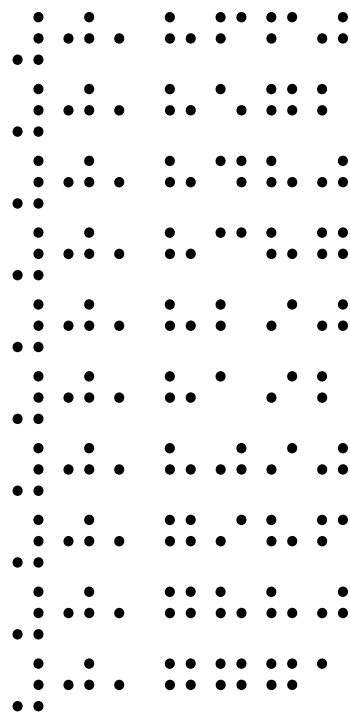
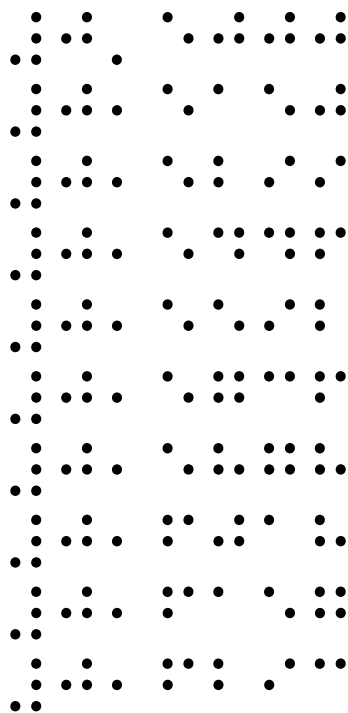
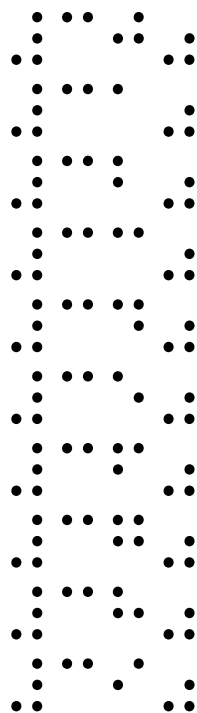
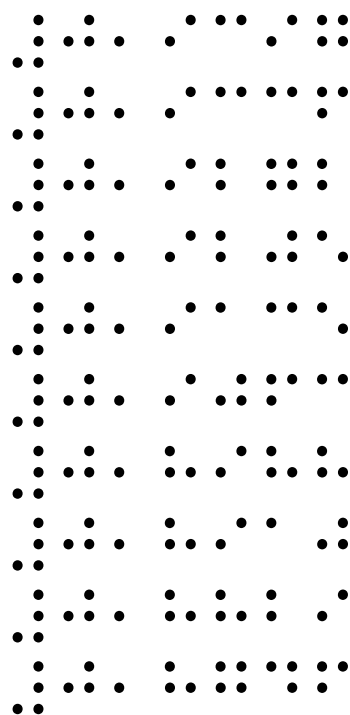
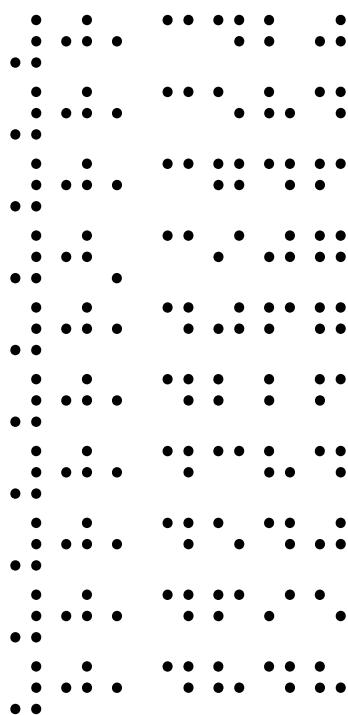
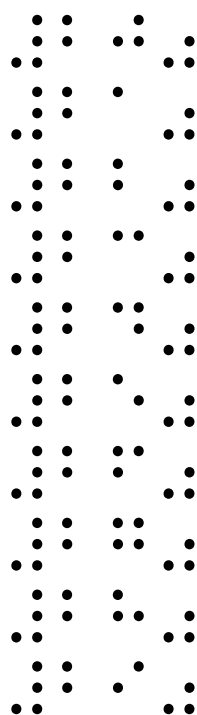
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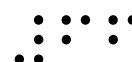
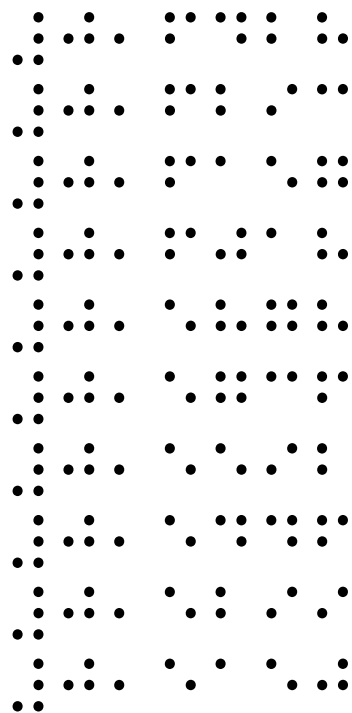
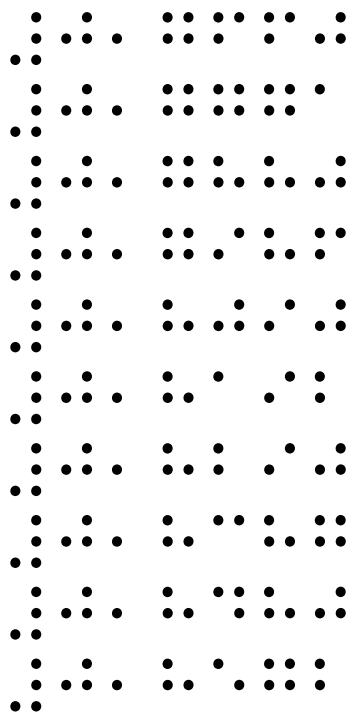
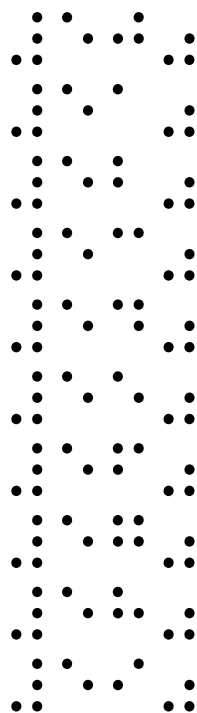
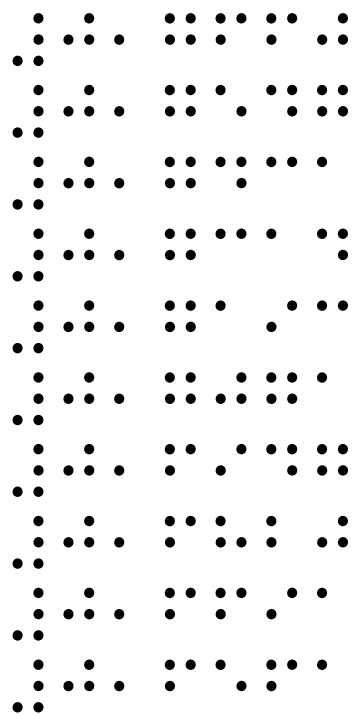
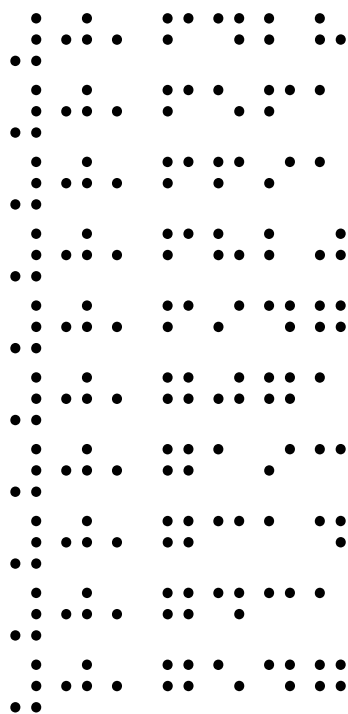
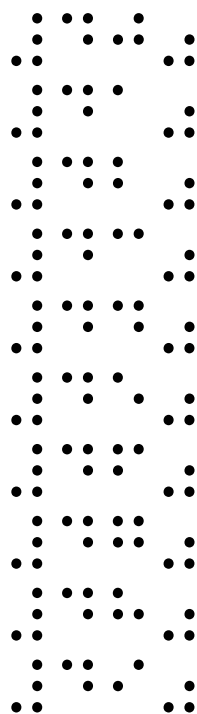
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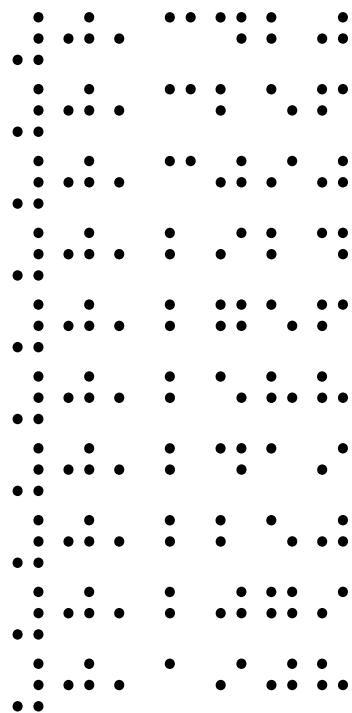
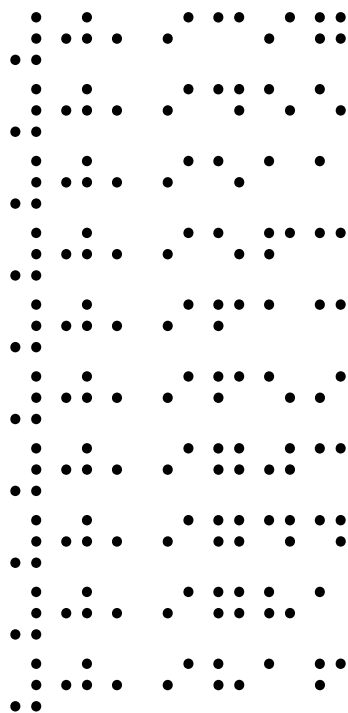
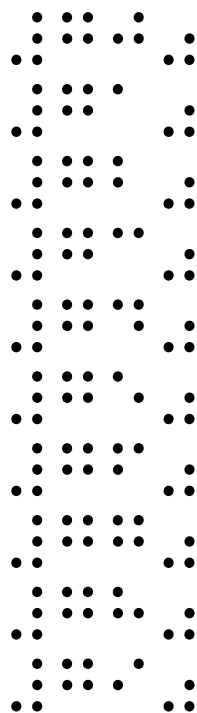
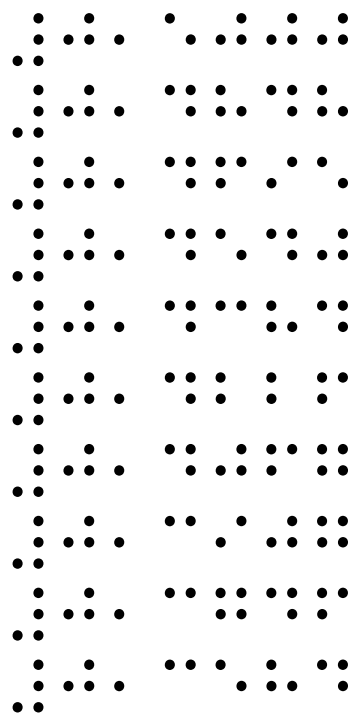
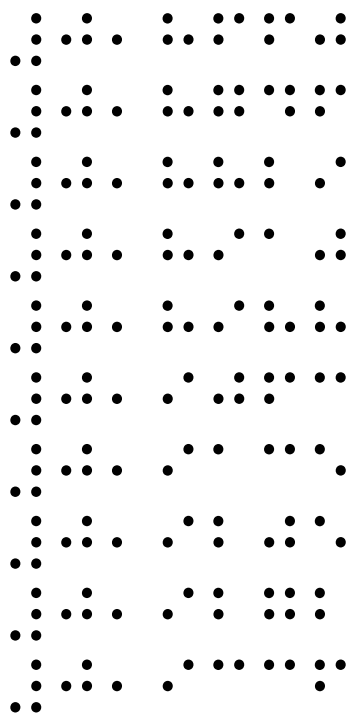
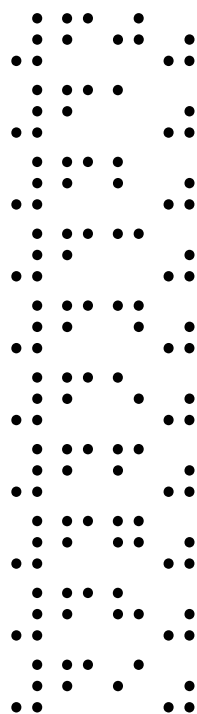
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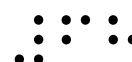
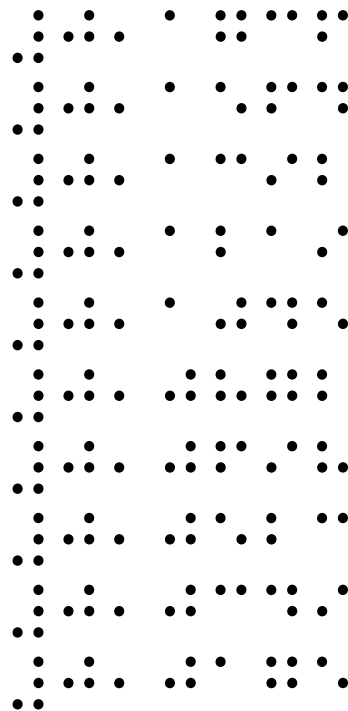
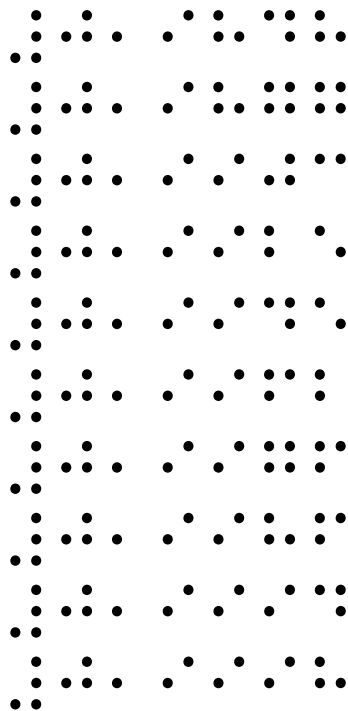
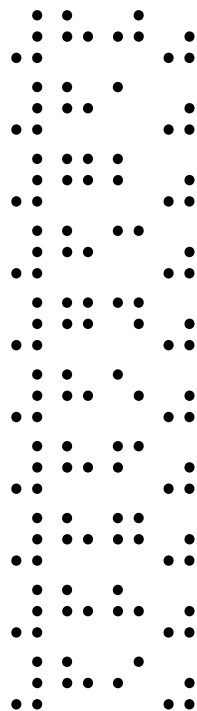
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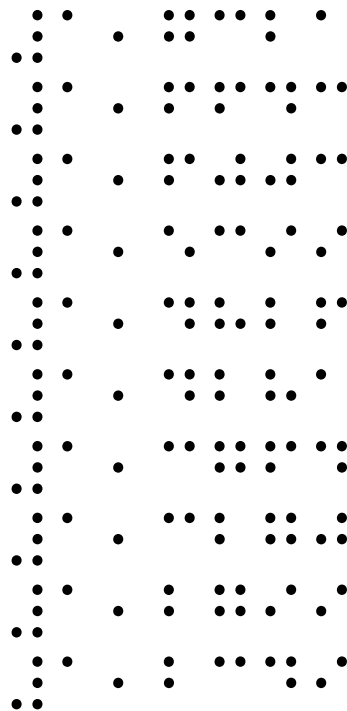
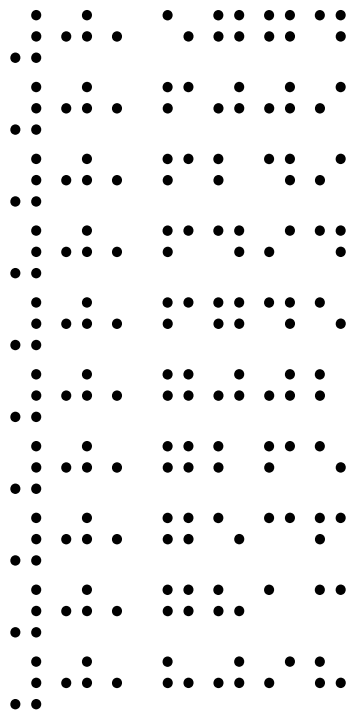
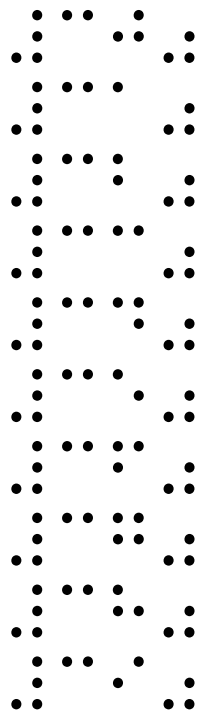
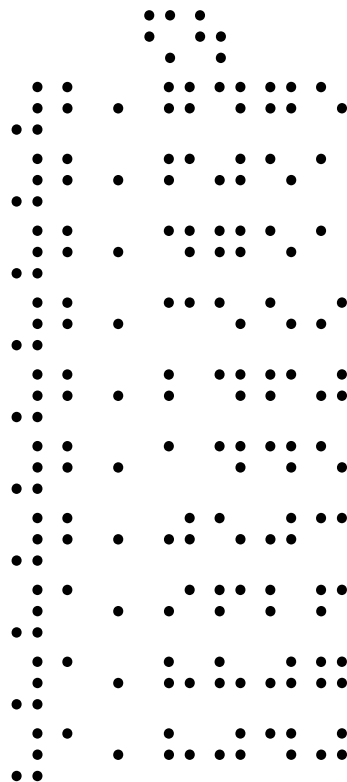
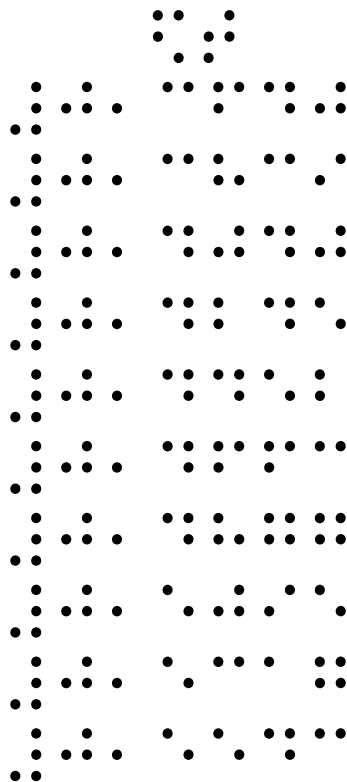
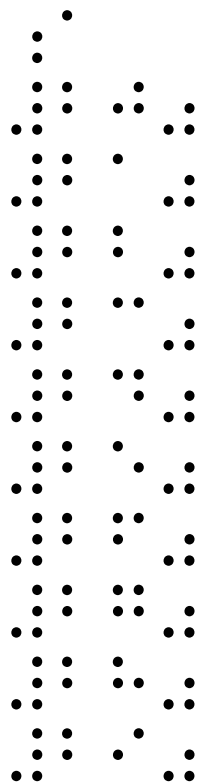
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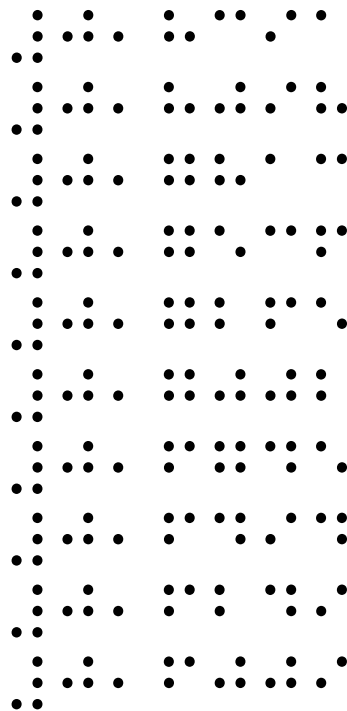
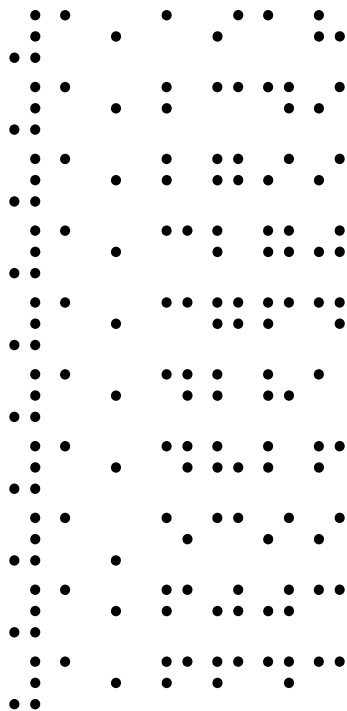
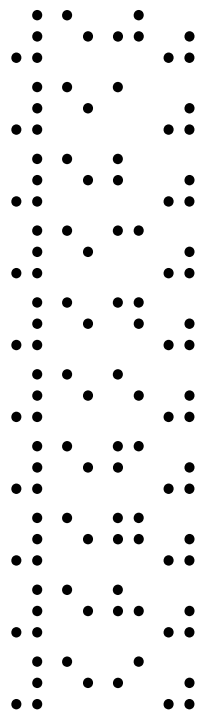
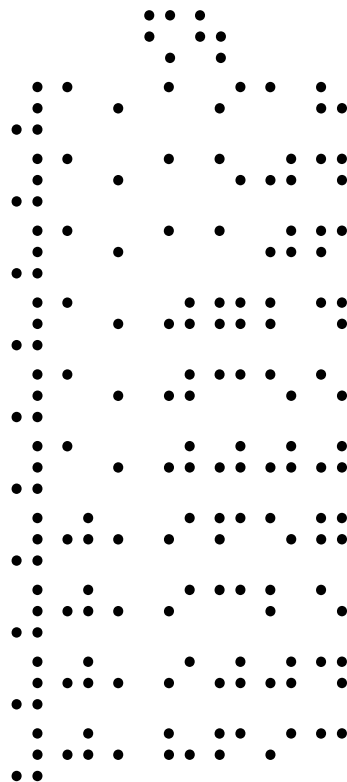
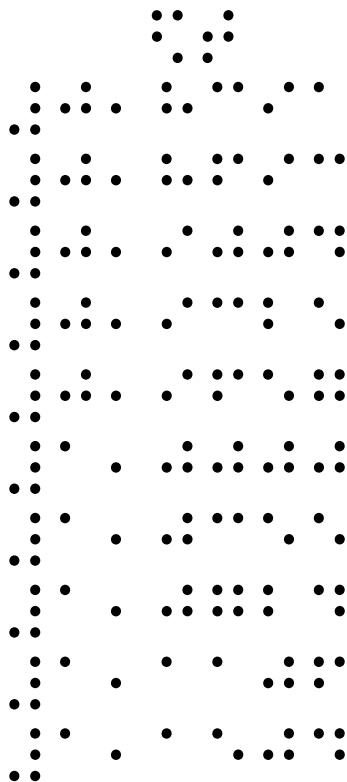
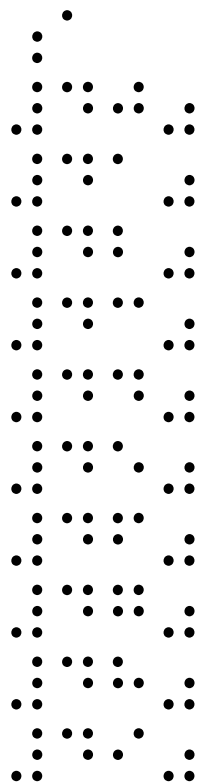
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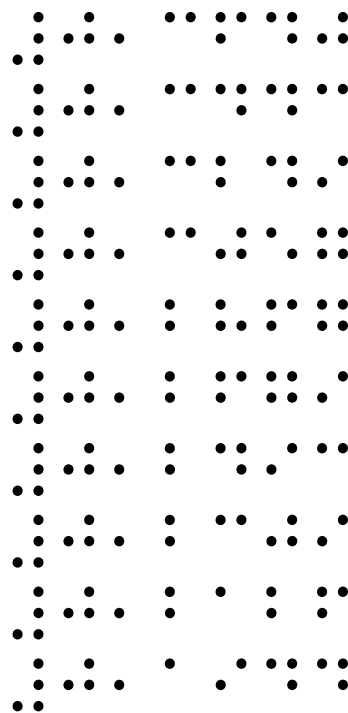
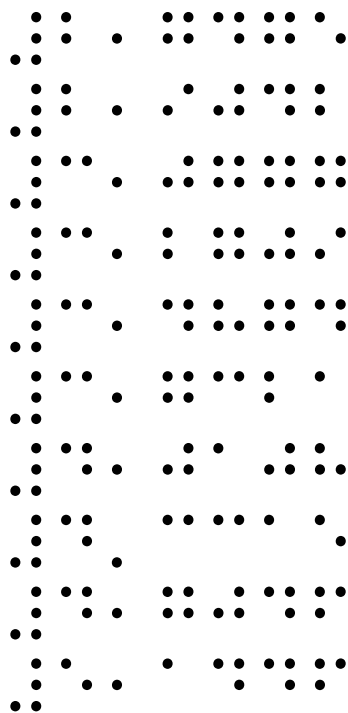
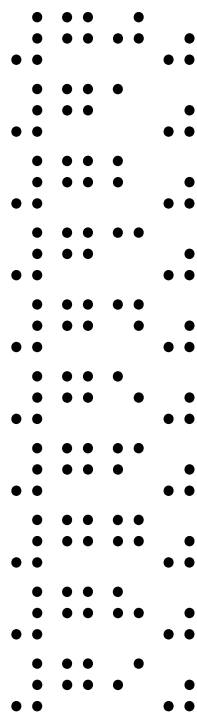
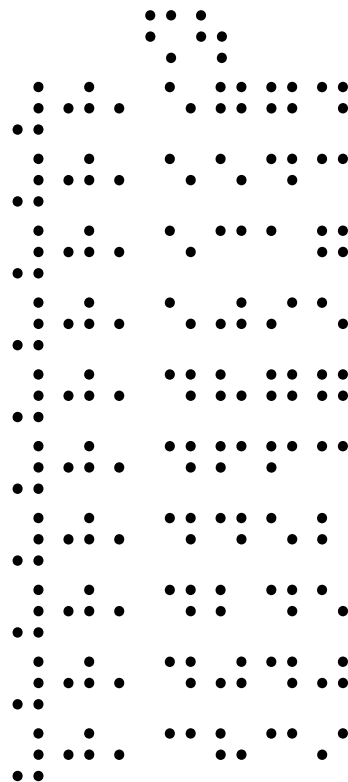
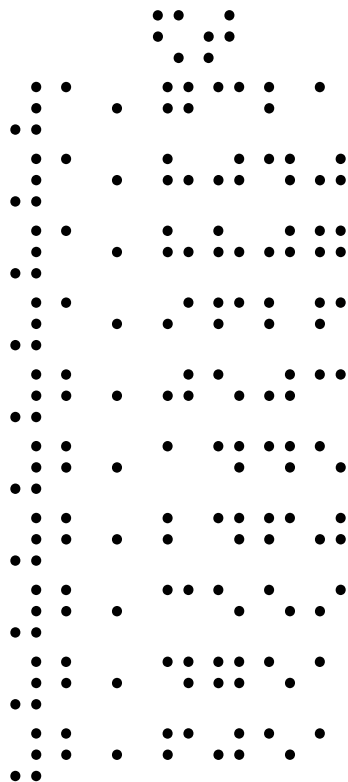
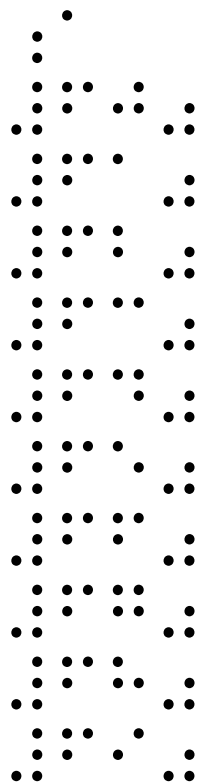
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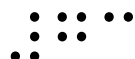
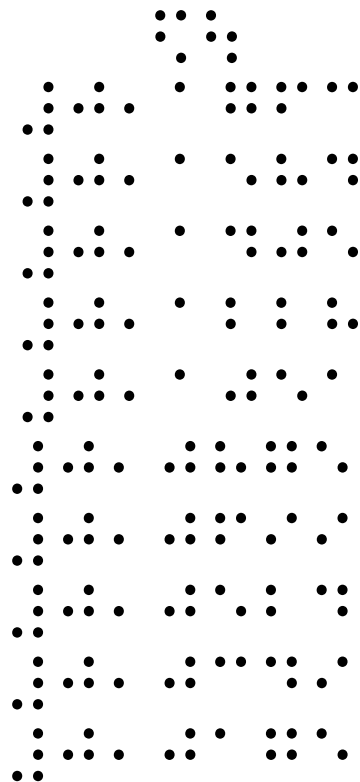
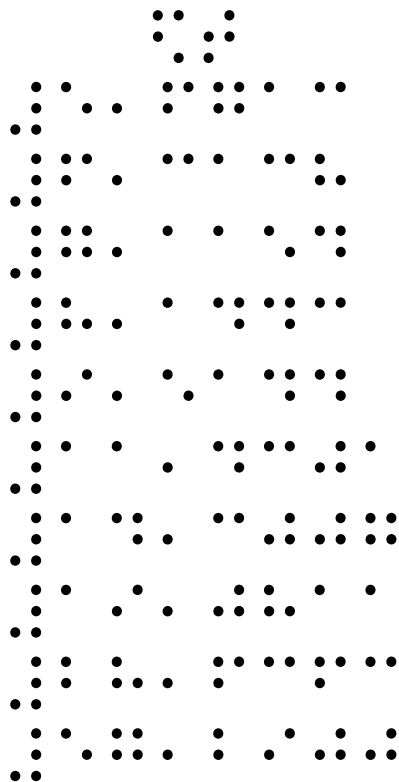
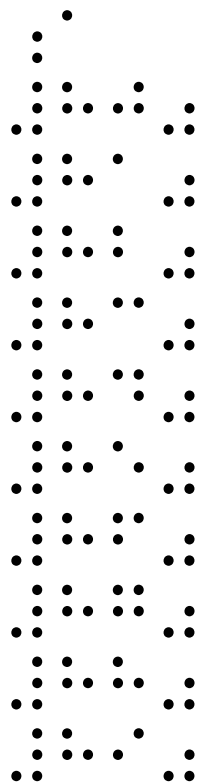
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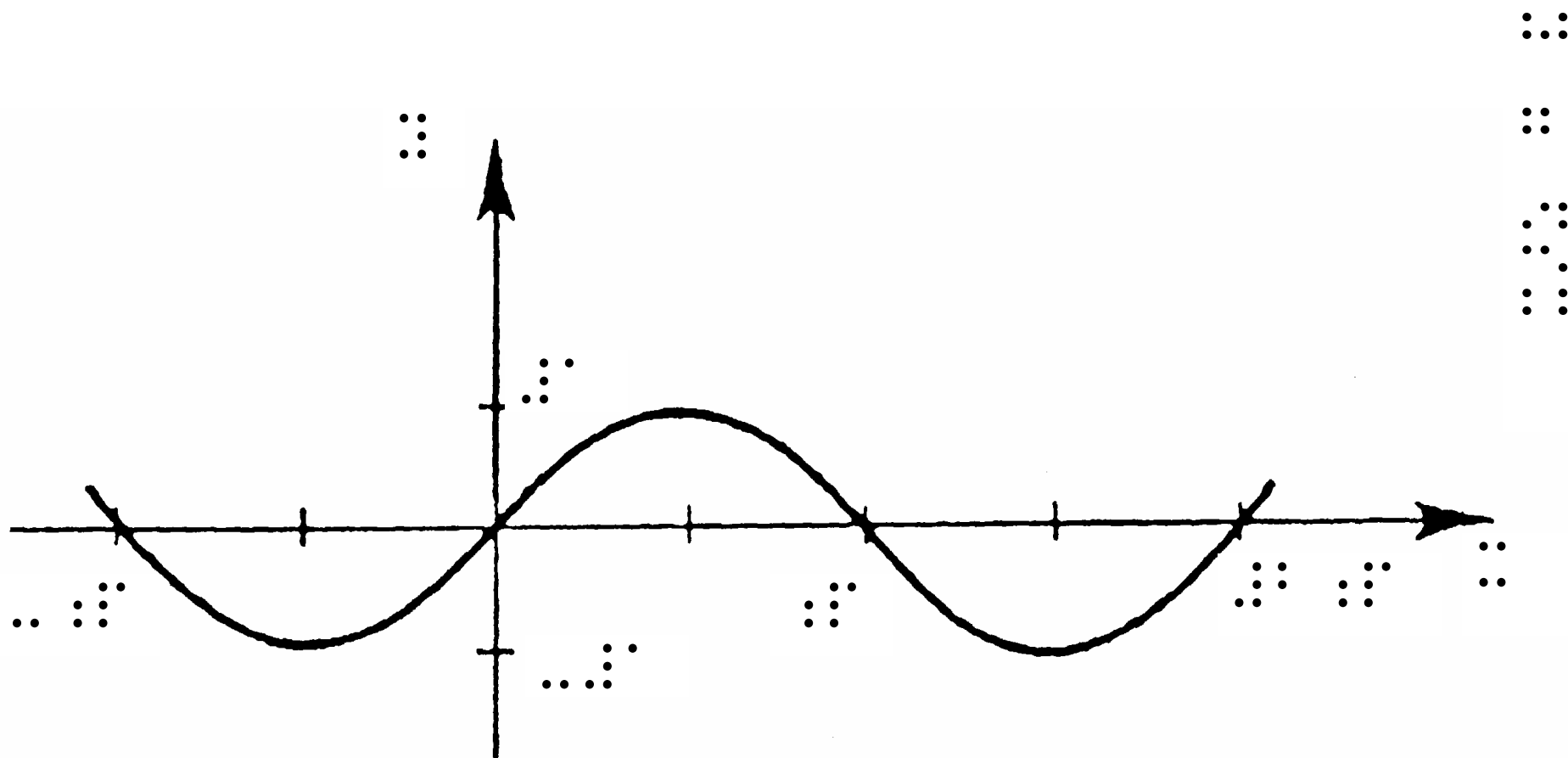
THE



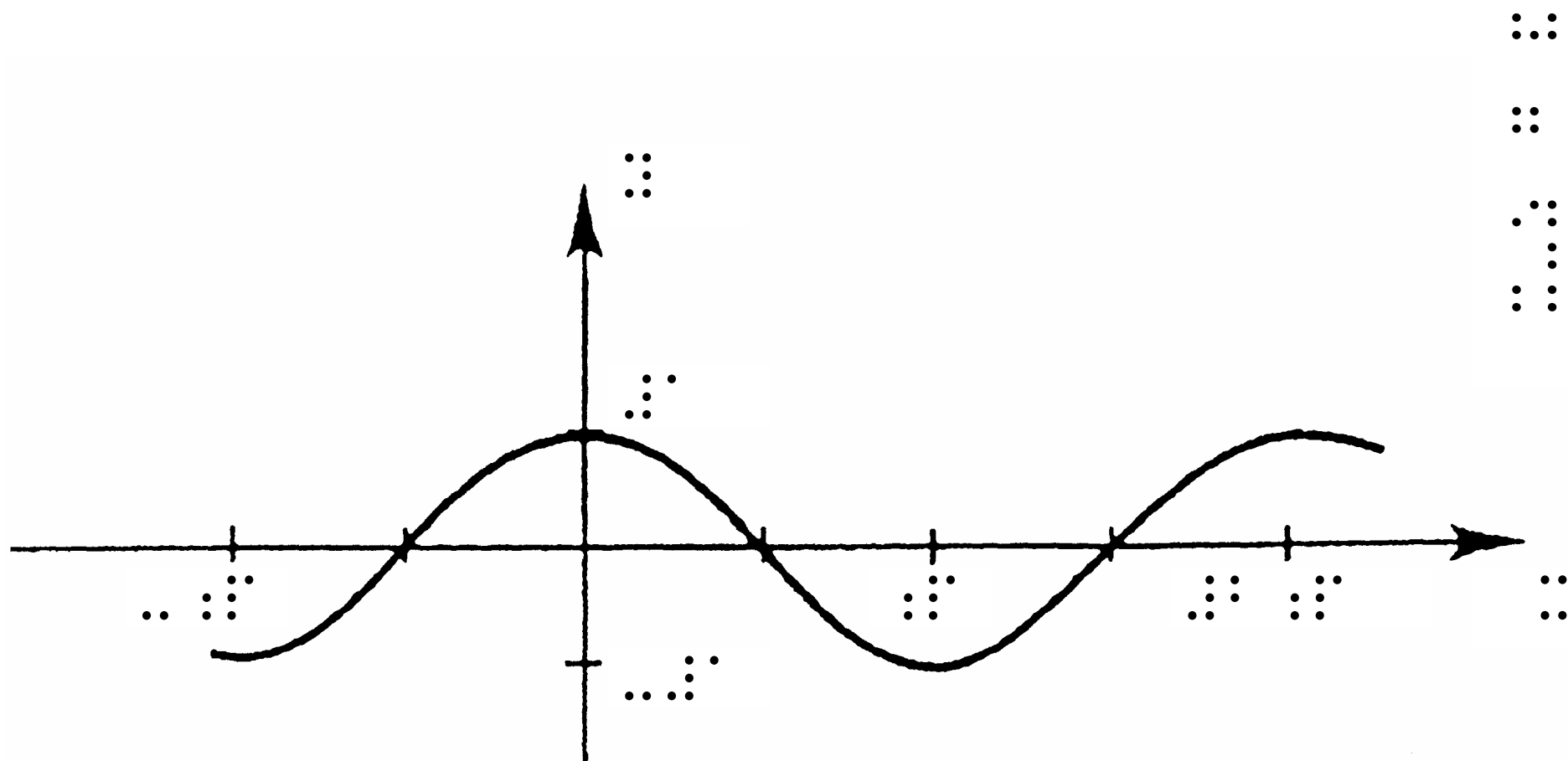




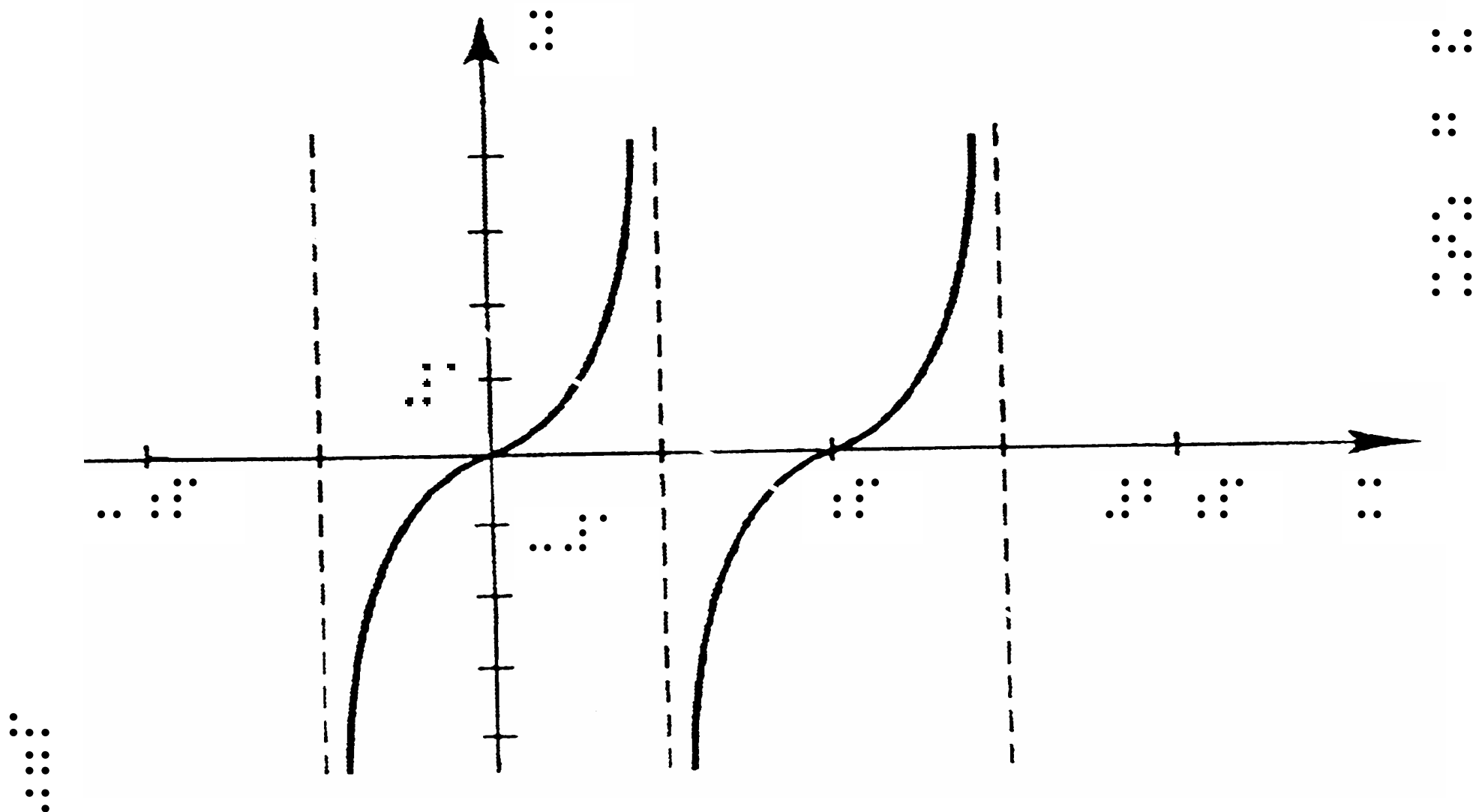


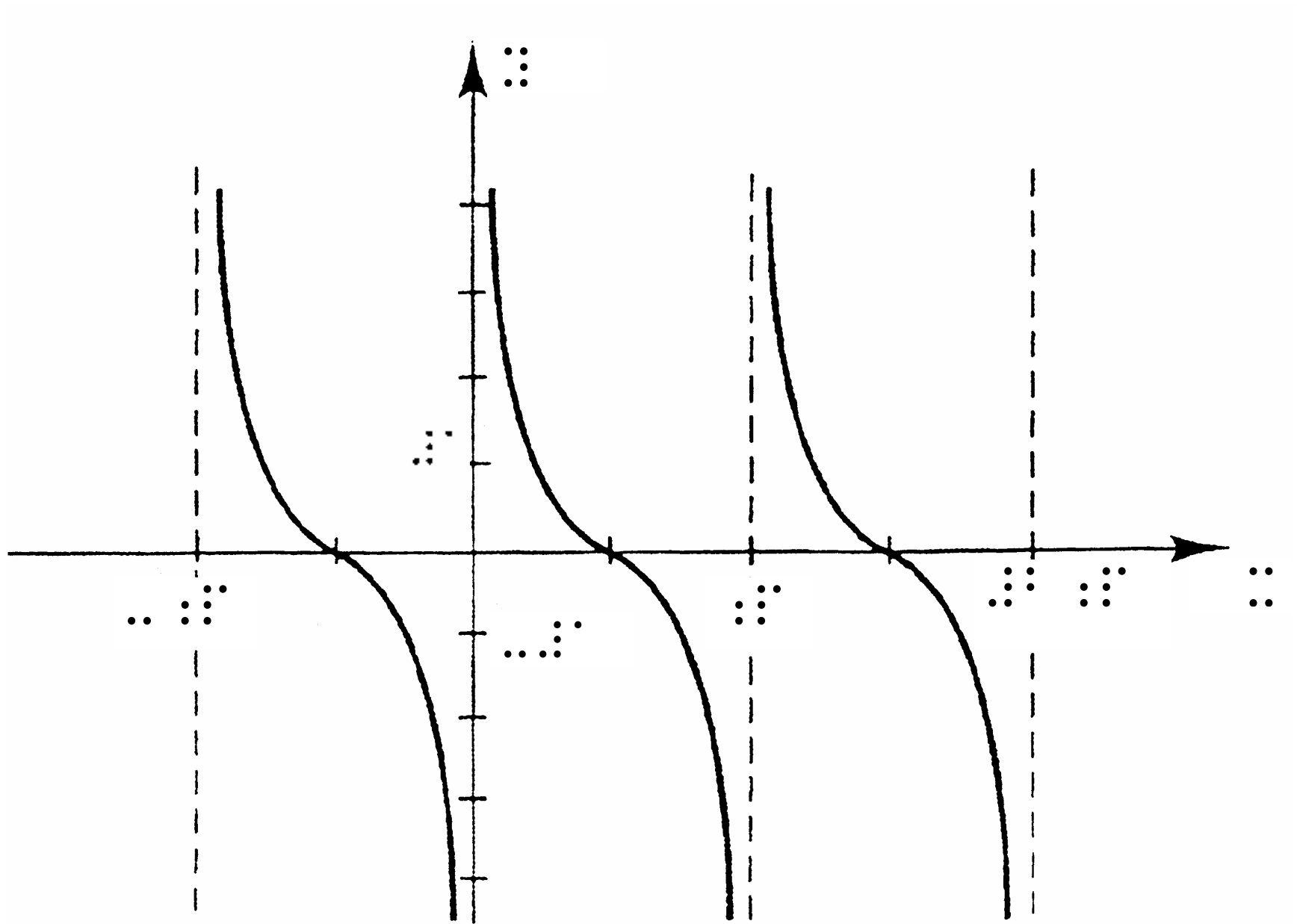


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Braille label: $y = 1$





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